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CHAPTER 1

REAL ANALYSIS

Unit 1.1: Point Set Topology

[CUET PG]

[CUET PG 2022]

1. Let A and B be two non-empty subsets of R. If $\underline{A}, A^{\circ}$, and A' denote closure, interior and set of limit points of A respectively, then

(1) $A^{\circ} \cap \underline{B}$ is open
(2) $A^{\circ} \cap B'$ is open
(3) $\underline{A} \cap B'$ is closed
(4) $\underline{A} \cap B$ is closed

Answer Key
1
3

[CU-CET]

[CUCET 2016]

1. Consider the set $S = \left\{x \in R : \frac{2x+1}{x+2} < 1\right\}$,where R is the set of reals. Determine which one of the following statements about S is correct.

(1) S is bounded below but not above and $inf S = -2$
(2) S is bounded above but not below and $sup S = 1$
(3) S is bounded both below and above with $inf S = -2, sup S = 1$
(4) S is neither bounded below nor above

[CUCET 2016]

2. Consider the set $S = \left\{\frac{mn}{1+m+n} : m, n \ are \ natural \ numbers\right\}$. Then determine which one of the following statements is correct.

(1) S is a bounded set
(2) S is bounded below with $inf S = \frac{1}{3}$ but not bounded above
(3) S is bounded above with $sup S = 1$ but not below
(4) S is neither bounded below nor above

[CUCET 2016]

3. Let $X = \{x \in R : x \neq 1, 2, \ldots, 100\}$ be a subset of R. Determine which one of the following statements is true

(1) Integers 1, 2,...,100 are the only limit points of X
(2) No limit point of X lies between 1 and 100
(3) X is a closed subset of R
(4) X is an open subset of R

[CUCET 2016]

4. Let $X = \left\{\frac{1}{n} : n \in Z, n \neq 0\right\}$be a subset of R (Z is the set of all integers). Determine which one of the following properties of X is true.

(1) X is a bounded set
(2) X is an open subset of R
(3) X is a closed subset of R
(4) X has no limit point in R

[CUCET 2016]

5. The set Z of all integers

(1) is an open subset of R
(2) is a closed subset of R
(3) is a compact subset of R
(4) has infinitely many limit points in R

[CUCET 2016]

6. Determine which one of the following subsets of R is connected.

(1) $(-\infty, 0) \cup (0, \infty)$
(2) The set Q of all rational numbers
(3) $\cup_{n=1}^{\infty}(-n, n)$
(4) $R \ Z$

[CUCET 2016]

7. Which one of the following statements is true

(1) Every closed interval in R is homeomorphic to R
(2) Every open interval of the type (a, b) is homeomorphic to R
(3) Every interval of the type $[a, b)$ is homeomorphic to R
(4) Every interval of the type $(a, b]$ is homeomorphic to R

[CUCET 2017]

8. The set $U = \left\{x \in R : sin \ sin \ x = \frac{1}{2}\right\}$ is

(1) open
(2) closed
(3) both open and closed
(4) neither open nor closed

[CUCET 2021]

9. Let Q denotes the complement of the set of rational numbers Q. The set of all boundary points of Q in the set of real numbers R is

(1) Q^c
(2) R
(3) ϕ
(4) Q

[CUCET 2021]

10. The set $U = \{x \in R \mid sinx = \frac{1}{2}\}$ is
    (1) closed
    (2) open
    (3) neither open nor closed
    (4) both open and closed

Answer Key

1	2	3	4	5	6	7	8	9	10
3	2	4	1	2	3	2	2	2	1

[PRACTICE QUESTIONS]

1. Let A be a closed subset of $\mathbb{R}, A \neq \phi, A \neq \mathbb{R}$. Then A is
   (1) the closure of the interior of A.
   (2) a countable set.
   (3) a compact set.
   (4) not open.

2. Let A and B be two sets having m and n many elements respectively, where $m < n; m, n \in \mathbb{N}$. The number of injective functions possible from A to B is:
   (1) m^n
   (2) ${}^nP_m$
   (3) ${}^nC_m$
   (4) n^m

3. The set of rational numbers has:
   (1) Least upper bound property
   (2) Greatest lower bound property
   (3) Archimedean property
   (4) None of the above

4. Which of the following statement is not true?
   (1) $\sqrt{2}$ is an irrational number
   (2) $\sqrt{2}$ is not an algebraic number
   (3) $\sqrt{2}$ is not a transcendental number
   (4) $\sqrt{2}$ is not a rational number

5. Which set is uncountable?
   (1) The set of positive primes
   (2) The set of integers
   (3) The set of rational numbers
   (4) The set of irrational numbers in [0, 1]

6. An algebraic number is a root of a polynomial whose coefficients are rational. The set of algebraic number is
   (1) finite
   (2) countably infinite
   (3) uncountable
   (4) none of these

7. Which one of the following does not imply $= 0$?
   (1) For all $\varepsilon > 0, 0 \leq \alpha < \varepsilon$
   (2) For all $\varepsilon > 0, -\varepsilon < \alpha < \varepsilon$
   (3) For all $\varepsilon > 0, \alpha < \varepsilon$
   (4) For all $\varepsilon > 0 - \varepsilon, < \alpha < \varepsilon$

8. Let S and T be subsets of $\mathbb{R}$. Select the incorrect statement:
   (1) $(\text{int } S) \cap (\text{int } T) = \text{int}(S \cap T)$
   (2) $(\text{int } S) \cup (\text{int } T) \subset \text{int}(S \cup T)$
   (3) $\overline{S}$ is closed in $\mathbb{R}$
   (4) $\overline{T}$ is the largest closed set containing T

9. Choose the correct statements
   (1) $\cup_{n=1}^{\infty} [1/n, 2] = [0,2]$
   (2) $\cup_{n=1}^{\infty} [1/n, 2] = (0,2]$
   (3) $\cap_{n=1}^{\infty} (1/n, 2] = (1,2]$
   (4) $\cap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2\right] = (1,2)$

10. If $\mathbb{Q} \subset A \subset \mathbb{R}$, which of the following must be true?
    (1) If A is open, then $A = \mathbb{R}$
    (2) If A is closed, then $A = \mathbb{R}$
    (3) If A is uncountable, then A is closed
    (4) If A is countable, then A is closed

11. Which one of the following set is not countable?
    (1) $\mathbb{N}^r$, where $r \geq 1$ and $\mathbb{N}$ is the set of natural numbers
    (2) $\{0,1\}^{\mathbb{N}}$, the set of all the sequences which takes values 0 and 1.
    (3) $\mathbb{Z}$, set of integers
    (4) $\sqrt{2}\mathbb{Q}$, $\mathbb{Q}$ is set of rational numbers.

12. Let X be a countably infinite subset of $\mathbb{R}$ and A be a countably infinite subset of X. Then the set $X \backslash A = \{x \in X | x \notin A\}$
    (1) is empty
    (2) is a finite set.
    (3) can be a countably infinite set.
    (4) can be an uncountable set

13. The subset $A = \{x \in \mathbb{Q} : x^2 < 4\}$ of $\mathbb{R}$ is
    (1) bounded above but not bounded below
    (2) bounded above and sup $A = 2$.
    (3) bounded above but does not have a supremum.
    (4) not bounded above.

14. The subset $A = \{x \in \mathbb{Q} : -1 < x < 0\} \cup$ of $\mathbb{R}$ is
    (1) bounded, infinite set and has a limit point in $\mathbb{R}$
    (2) unbounded, infinite set and has a limit point in $\mathbb{R}$
    (3) unbounded, infinite set and does not have a limit point in R
    (4) bounded, infinite set and does not have a limit point in $\mathbb{R}$

15. Let
    $S = \prod_{n=1}^{\infty}\left[-\frac{1}{2}, 1+\frac{1}{n}\right]$
    then S equals [DU-2017]
    (1) [0,1] (2) (0,1]
    (3) (0,1) (4) [10,1)

16. Consider the sets
    $S = \left\{\frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime}\right\} T = \{x^2 : x \in \mathbb{R}\}$
    Then
    (1) $sup(S \cap T) = 1$
    (2) $\sup S = 1$ and $\inf \ \mathrm{T} = 0$
    (3) $\sup S = \frac{1}{2}$ and $\inf \ \mathrm{T} = 0$
    (4) $\inf(S \cup T) = \frac{1}{2}$

17. Which of the following sets is in one-to-one correspondence with $\mathbb{N}$
    (I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$
    (II) $\{\dots 3, -3, -2, -1, 0, 1, 2, 3, \dots\}$
    (III) $\left\{\frac{P}{q} : p, q \in \mathbb{N}, q \neq 0\right\}$
    (IV) $\left\{\frac{P}{q} : p, q \in \mathbb{N}\right\}$
    (1) (I) and (II) (2) (I), (II) and (II)
    (3) (I) and (IV) (4) All of above

18. Let $X = \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{Q}, y \in \mathbb{R} \backslash \mathbb{Q})$. Where $\mathbb{Q}$ is the set of rationals. Then
    (1) X is an open and dense subset of $\mathbb{R}^2$
    (2) X is an open but not dense subset of $\mathbb{R}^2$
    (3) X is not an open but a dense subset of $\mathbb{R}^2$
    (4) X is neither an open nor a dense subset of $\mathbb{R}^2$

19. Let $S = \{x \in \mathbb{R} : 3 - x^3 > 0\}$. Then
    (1) S is bounded above and 3 is the least upper bound of S.
    (2) S is bounded above and does not have a least upper bound in $\mathbb{R}$
    (3) S is bounded above and does not have a least upper bound in Q , the set of rational members.
    (4) S is not bounded above.

20. Let P be the set of all the polynomials with rational coefficients and S be the set of all sequences of natural numbers. Then which one of the following statement is true?
    (1) S is countable but is not
    (2) Both the sets P and S are uncountable
    (3) Both the sets P and S are countable.
    (4) P is countable but S is not

21. Let $S = \cap_{n=1}^{\infty}\left[2-\frac{1}{n}, 3+\frac{1}{n}\right]$.
    Then S equals
    (1) (2,3] (2) [2,3]
    (3) [2,3) (4) (2,3).

22. Let $S == \cap_{n=1}^{\infty}\left[0, \frac{1}{n}\right]$ then
    (1) $\limsup a_n = +\infty, \liminf a_n = -1$
    (2) $\limsup a_n = +\infty, \liminf a_n = 0$
    (3) $\limsup a_n \neq +\infty, \liminf a_n = -\infty$
    (4) $\limsup a_n = 1, \liminf a_n = -1$

23. Let $S = \cap_{n=1}^{\infty}\left[0, \frac{1}{n}\right]$. Then which one of the following statement is true?
    (1) $\inf S > 0$
    (2) $\sup S = 1$ and $\inf S = 0$
    (3) $\sup > 0$
    (4) $\sup S = \inf S = 0$

24. Let $S = \{x \in R \backslash x^2 \leq 5\} \cap \mathbb{Q}$ which of the following statements is true about S ?
    (1) Sis bounded above and $sup, S \in \mathbb{Q}$
    (2) S is bounded above and . $sup\ S$
    (3) S is a closed interval.
    (4) S is an open interval

25. The set of all real numbers x for which there is some positive real number y such that $x < y$ equal to
    (1) $\mathbb{R}$
    (2) The set of all negative real numbers
    (3) $\{0\}$
    (4)None of the above

26. Let $X \subset N$ be a nonempty finite set and$\subset N$ be an infinite set. Define $A = \{x - y : x \in X \text{ and } y \in Y\}$. Then
    (1) $\inf(A) = -\infty$ and $\sup(A) = \infty$.
    (2) $\inf(A) = -\infty$ and $\sup(A) < \infty$.
    (3) $\inf(A) > -\infty$ and $\sup(A) = \infty$.
    (4) $\inf(A) = -\infty$ and $\sup(A) < \infty$.

27. Let X be a nonempty set and $A, B \subset X$. If $(A \cup B) \backslash (A \cap B)$ is a finite set, then,
    (1) Both A and B are finite sets.
    (2) Atleast one of A, B is a finite set
    (3) X is a finite set
    (4) None of the above

28. Let X be a set with 2m+1 elements, where $m \geq 4$. The number of subsets of X having at least $m + 1$ elements is
    (1) $2^{2m} + 1$
    (2) $2^{2m} - 1$
    (3) 2^{2m}
    (4) $2^m + 1$

29. Let $A \Delta B$ denote the symmetric difference of two sets : $A \Delta B := (A \backslash B) \cup (B \backslash A)$ which of the following is true.
    (1) $A \cap (B \Delta A) = \emptyset$ *alway hold true*
    (2) $(A \cup (A \Delta B)) \cap B \neq B$ *always hold true* .
    (3) $A \Delta B = (A \cup B) \cap (A^c \cap B^c)$ *always holds true*
    (4) None of the above is true

30. For subsets T_1 and T_2 of $\mathbb{R}$,
    we define $T_1 + T_2 := \{t_1 + t_2 | t_1 \in T_1, t_2 \in T_2\}$.
    Let
    $S_1 := (\mathbb{Q} \cap [0,1]) + \{\sqrt{2}\}$ and
    $S_2 := (\mathbb{Q} \cap [1,2]) + \{\sqrt{3}\}$. Let $S = S_1 \cup S_2$.
    Which one of the following is true?
    (1) There exists a one to one map from [0,1] to S
    (2) There exists an onto map from S to [0,1]
    (3) $\neq S_1 = \# S_2$
    (4) None of the above

31. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous map and $S = \left\{\pi + \frac{(-1)^n}{2n+1} | n \in \mathbb{N}\right\} \cup (0,1)$. Which one of the following is necessarily true?
    (1) $f((0,1))$ is an open set
    (2) $f(\bar{S})$ is compact
    (3) $f(S)$ is compact
    (4) $f(S)$ is closed but not compact

32. Which of the following is a compact subset of $\mathbb{R}$?
    (1) $\left\{-\frac{1}{n} : \in \mathbb{N}\right\} \cup \{0\} \cup (0, -]$
    (2) $\left\{-\frac{1}{n} : \in \mathbb{N}\right\} \cup (0, -]$.
    (3) $\{1,2,3,\} \cup \left\{6 + \frac{1}{n} : n \in \mathbb{N}\right\}$.
    (4) $\left\{-\frac{1}{n} : \in \mathbb{N}\right\} \cup (0,2]$

33. Which of the following sets has/have the same cardinality as $\mathbb{R}$?
    A. The set of all real sequences
    B. The set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$
    C. The set of all functions from $\mathbb{R}$ to the two-element set $\{0,1\}$
    (1) A only
    (2) B only
    (3) A and B only
    (4) C only

34. Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a continuous function such that $f(\alpha v) = f(v)$ for all $\alpha \in \mathbb{R}, \alpha > 0$ and all $v \in \mathbb{R}$.
    Consider the following statements:
    A. The image of f is compact.
    B. The image of f is uncountable.
    Which of the following is correct?
    (1) A is true but B is false.
    (2) B is true but A is false.
    (3) Both A and B are true.
    (4) Both A and B are false.

35. for any pair of subsets C and D of $\mathbb{R}^2$,let C^0 denote the interior of C, and C/D denote the set of points in C which do not belong to D. Consider the following statements:
    A. $\left(\frac{C}{D}\right)^0 = C^0/D^0$
    B. $\left(\frac{C^0}{D}\right)^0 = C^0/D^0$
    Which is the following is necessarily correct?

(1) A is true but B is false
(2) B is true but A is false
(3) Both A and B are true
(4) Both A and B are false.

36. Consider the subset $S := \{(x, y) \in \mathbb{R}^2 : y = x \, or \, |x| + |y| = 1\}$ of $\mathbb{R}^2$. Consider the following assertions about S.
A. S is a reflexive relation on $\mathbb{R}$.
B. S is a transitive relation on $\mathbb{R}$.
C. There exists a surjective map from $\mathbb{R}$ onto S.
D. There exists an injective map from S into $\mathbb{Q}^3$.
Which of the above assertions is/are correct?
(1) A only (2) A and D only
(3) A and C only (4) A and B only`

37. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\} \subset \mathbb{R}$.
Which of the following assertion about S is correct
(1) For any two maps $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}$ and $g : \mathbb{R} \to S$, their compositions $g \, 0 \, f$ is never surjective
(2) There exists a surjective map from S onto R^2/Z^2
(3) There exists an injective map from R/Q into S.
(4) For any two maps $f : S \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^3$, their composition $g \, 0 \, f$ has a countable range.

38. Consider the following
A. Union of any collection of open sets in $\mathbb{R}^n$ is open
B. Intersection of any collection of open sets in R^n is open. Then
(1) B is true and A is false.
(2) A is true and B is false.
(3) Both A and B are true.
(4) both A and B are false.

39. Suppose $\mathcal{R}$ Relation defined on $\mathbb{N}_{by}(a, b) \in \mathcal{R}$- iff $a - b$ is a multiple for 4 or a multiple of 6. Consider the statements:
A. $\mathcal{R}$ is symmetric.
B. $\mathcal{R}$ is transitive.
C. There is an infinite subset $U \subset \mathbb{N}$ or $\mathbb{N}$ such that $\mathcal{R}$ defined on U is an equivalence relation. Then the number of correct statements above is
(1) 1 (2) 2
(3) 3 (4) 0

Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
4	2	3	2	4	2	3	4	2	2
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	3	2	2	1	3	4	3	2	4
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
2	2	4	3	4	2	4	3	4	3
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	
2	1	3	1	1	4	4	2	2	

EXPLANATIONS

[CUET PG]

1. **Explanation:**

Let A & B be two non-empty subject of $\mathbb{R}$.

Let

$A = Q \qquad B = Q$

$\bar{A} = \mathbb{R} \qquad A^o = \phi \qquad B' = \mathbb{R}$

$\bar{B} = \mathbb{R}$

1. $\bar{A} \cup B^\circ = \mathbb{R} \cap Q = Q$

Neither open nor closed (4) incorrect.

2. $A = [a \, b] B = [a, b]$

$A^\circ = (a, b) \qquad B^\circ = (a, b)$

$\bar{A} = [a, b] \qquad \bar{B} = [a, b]$

$A^\circ \cup B' = [a, b]$- closed set.

So (2) incorrect.

$A = [0, 2] \qquad B = [1, 3]$

$A^\circ = (0, 2) \qquad \bar{B} = [1, 3]$

$B' = [1, 3]$

(1) $A^\circ \cap \bar{B} = [1, 2)$

Option A incorrect.

(2) $A^\circ \cup B = (0, 3]$

2 incorrect.

3 correct.

[CU-CET]

1. **Explanation:**

$S = \left\{x \in \mathbb{R} \frac{2x+1}{x+2} < 1\right\}$

Take $x = 1$

$\frac{2+1}{3} = 3 < 1$ which not satisfy given condition

Take, $x = 0 \qquad \frac{1}{2} < 1$

$x = -1 \qquad \frac{-1}{1} < 1$

$x = -3 \qquad \frac{-6+1}{-1} = 5 < 1$

which not satisfy given condition

$-2 < x < 1$

$\therefore$ S is bounded both below & above with $infs = -2$

$sups = 1$

$\therefore$ option (3) is correct

2. **Explanation:**

$S = \left\{\frac{mn}{m+n+1}, m, n \in \mathbb{N}\right\}$

$x = \frac{mn}{m+n+1}$

$\Rightarrow x + 1 = \frac{(m+1)(n+1)}{1+m+n}$

$= \frac{(m+1)}{m}\frac{(n+1)}{n}$

$= \frac{x+1}{x}$

Now, note that for positive integer k,

we have $1 < \frac{k+1}{k} \leq 2$

We can thus infer $\frac{x+1}{x}$ takes value bet. 1 and 4

i.e. $1 < \frac{x+1}{x} \leq 4$

$\Rightarrow 0 < \frac{1}{x} \leq 3$

$\Rightarrow x \in \left[\frac{1}{3}, \infty\right)$

We see that $infs = {}^1/_3$ & $sups = \infty$

$\Rightarrow$ option (2) is correct

3. **Explanation:**

Let $X = \{x \in \mathbb{R}, x \neq 1,2,\ldots,100\}$ be subset of $\mathbb{R}$

$X = R - \{1,2,\ldots,100\}$

$X^c = \{1,2,3,\ldots 100\}$

$(X^c)' = \phi$

X^c is not closed

$\Rightarrow$ X is open

$\Rightarrow$ option (4) is correct

4. **Explanation:**

$X = \left\{\frac{1}{n} n \in \mathbb{Z}\, n \neq 0\right\} \leq \mathbb{R}$

Given set X has upper bound 1 & lower bound 0

$\therefore$ Hence given set is bounded set

$\Rightarrow$ option (1) is correct

5. **Explanation:**

$(n, n+1)$ if is open in $\mathbb{R}$ for any $n \in \mathbb{Z}$

$\therefore U = \cup_{n\in\mathbb{Z}}(n, n+1)$ is open in $\mathbb{R}$

(any union of open set is open)

Thus $\mathbb{R}|U$ is closed in $\mathbb{R}$

(a compliment of an open set is a closed set)

But $\mathbb{R}|U = \mathbb{Z}$

Thus $\mathbb{Z}$ is closed subset of $\mathbb{R}$

$\Rightarrow$ option (2) is correct

6. **Explanation:**

$\cup_{n=1}^{\infty}(-n, n)$ is subset of $\mathbb{R}$ is connected

$\Rightarrow$ option (3) is correct

7. **Explanation:**

Every open interval of type (a,b) is homomorphic to $\mathbb{R}$

$\Rightarrow$ option (2) is correct

8. **Explanation:**

$U = \left\{x \in \mathbb{R}: \sin x = \frac{1}{2}\right\}$

$\sin x: \mathbb{R} \to [-1,1]$ is continuous function

Now since $\left\{\frac{1}{2}\right\}$ is closed in $[-1,1]$

$U = \sin^{-1}\left(\frac{1}{2}\right)$

We get that

U is closed in $\mathbb{R}$

$\Rightarrow$ option (2) is correct

9. **Explanation:**

Set of boundary point of $\mathbb{Q}^C$ are $\mathbb{Q}^C$

10. **Explanation:**

Let $U = \left\{x \in \mathbb{R} \mid \sin x = \frac{1}{2}\right\}$

$\sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6} + 2\pi n \qquad n \in \mathbb{N} \cup \{0\}$

Neither open nor closed.

[PRACTICE QUESTIONS]

1. **Explanation:**

A is closed $A \subseteq \mathbb{R}\ A \neq \phi, A \neq \mathbb{R}$

A^0 – interior of A $\overline{A}$ – closure of A

(1) $A = \overline{A}^{\circ}$

$A = \{1] \cup [2,3]\ A \subseteq \mathbb{R}$ closed $A \neq \phi\ A \neq \mathbb{R}$

$A^{\circ} = (2,3)\ \overline{A}^{\circ} = [2,3]$

$A = \{1\} \cup [2,3] \neq [2,3] = \overline{A}^{\circ} \Rightarrow A \neq \overline{A}^{\circ}$

(2) $A = [1,2]\ A \neq \phi\ A \neq \mathbb{R}$ closed uncountable

(3) $A = [0, \infty)\ A \neq \phi\ A \neq \mathbb{R}$

Let $B \subseteq \mathbb{R}\ B$ is closed if B^c is open $A^c = (-\infty, 0) \rightarrow$ open set in $\mathbb{R}$.

$A \neq$ compact.

A is compact in $\mathbb{R}^n$ iff A is closed & bounded $(A \subseteq \mathbb{R}^n)$.

(4) $\therefore A$ is not open

Suppose A is open

$\mathbb{R} = A \cup A^c\ A \subseteq \mathbb{R}, A \neq \phi\ A -$ closed

$A \neq \phi \Rightarrow A^c \neq \mathbb{R}$

$A^c \neq \phi\ A^c \neq \mathbb{R}, A^c -$ open

$A \neq \phi\ A \neq \mathbb{R}\ A -$ open

A & A^c forms a seperation for $\mathbb{R}$.

$\therefore \Rightarrow \mathbb{R}$ is not connected.

A contradiction

⇒ A not a open

⇒ option (4) is correct.

2. **Explanation:** A and B be two set having m,n many element respectively, where $\mathrm{m} < \mathrm{n}$, $\mathrm{m,n} \in \mathbb{N}$. The number of injective function possible form A to B is

$\mathrm{F : A \to B}$

$|\mathrm{A}| = \mathrm{m} \qquad |\mathrm{B}| = \mathrm{n} \Rightarrow \mathrm{m} < \mathrm{n}$

Total injective (one – one) function is $\mathrm{nP_m}$

⇒ option (2) is correct

3. **Explanation:** The sett of rational numbers has Archimedean property

⇒ option (3) is correct.

4. **Explanation:**

$\therefore \sqrt{2}$ is an irrational number.

$\therefore \sqrt{2}$ is an algebraic number not transcender number.

$\therefore$ option (2) is correct.

5. **Explanation:**

⇒ The set of positive primes is countably infinite set ⇒ countable

⇒ Set of integer is countable

⇒ The set of rational number is also countable

⇒ The set of irrational number in [0,1] which is uncountable

⇒ Option (4) is correct

6. Let α be an algebraic number, then by definition, there exist a polynomial, having all coefficient rational, such that $p(\alpha) = 0$. In other words, all roots of a polynomial having all coefficient rational are algebenic number.

Let for a fixed integer n, put

$P_n = \{a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n; a_i \in \mathbb{Q}$ for all $i = 1,2,3 \ldots n\}$

Then the mapping,

$(a_0\, x^n + a_1\, x^{n-1} + \cdots + a_{n-1}\, x + a_n\,) \to (a_0, a_1, \ldots a)$

is P_n to $\mathbb{Q}^{n+1}$

Since, $\mathbb{Q}$ is countable, so is countable product $\mathbb{Q}^{n+1}$.

So set of all polynomials, having rational coefficient is $\cup_{n=0}^{\infty} P_n$, which is a countable union of countable sets, and so countable.

Since, a polynomial of degree n, have at most n roots, so the set of all possible roots of all polynomials with rational coefficients is a countable union of finite sets, hence at most countable.

So, the set of all algebraic numbers are countable.

Hence option (2) is correct

Since, $\varepsilon > 0$ was choosen arbitrary. Let $a = \frac{\varepsilon}{2} > 0\ as\ \varepsilon$ and $\frac{\varepsilon}{2} < \varepsilon,$

7. **Explanation:**

$\therefore$ for all $\varepsilon > 0$, we have $a\ < \varepsilon$, and $a\ \neq 0$.

Hence option (3) is correct

8. **Explanation:** Statement in option (D) is incorrect, as $\overline{T}$ is the smallest closed set which contain T

For example let $T = (0,1)$ is set in $\mathbb{R}$ and here $\overline{T} = [0,1]$ and $T \subset \overline{T}$

But this is not the largest closed set as $S = [10,8]$ is also closed set containing T.

(we take [0,8] as an example)

Hence option (4) is correct.

$$a_n = \begin{cases} 2^n + n^2 & ,n \leq 100 \\ 3 + (-1)^n \frac{n^2}{2^n+1} & ,n > 100 \end{cases}$$

Note that to check the convergence or divergence of a sequence, if we remove some finite number of terms, it wont have any effect on the convergence or divergence.

For large,n, $n^2 < 2^n + 1$, and $\lim\limits_{n\to\infty} \frac{n^2}{2^n+1} \left(\frac{\infty}{\infty} \text{from}\right)$

Applying L'Hopital Rule

$$\lim_{n\to\infty} \frac{n^2}{(\log 2).2^n} \left(\frac{\infty}{\infty} \text{from}\right)$$

$$\lim_{n\to\infty} \frac{n^2}{2^n.(\log 2)^2} \left(\frac{\infty}{\infty} \text{from}\right)$$

$$\lim_{n\to\infty} a_n = 3$$

i.e., $\langle a_n \rangle$ is convergent sequence, hence cauchy.

Hence option (1) is correct

9. **Explanation:**

For option (A): Note that for all $n \in \mathbb{N}, \frac{1}{n} > 0$

Therefore, $0 \notin \bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right]$

$\Longrightarrow 0 \notin \bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right]$

$\bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right] \neq [0,2]$

For option (B): Let $x \in (0,2]$ be arbitrary, then $0 < x$, then there exist a natural number n such that $0 < \frac{1}{n} < x$,

for all $x > 0$, we have $n \in \mathbb{N}$ such that $(x, 2] \subset \left(\frac{1}{n}, 2\right]$

Since $x > 0$ was arbitrary

we have $(0,2] \subseteq \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right]$ .....(1)

If $y \in \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right]$ is arbitrary, then $y \in \left(\frac{1}{n}, 2\right]$ fore some $n \in \mathbb{N}$

i.e. $y > \frac{1}{n} > 0$

$\therefore \quad \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right] \subseteq (0,2]$ .....(2)

From (1) and (2),

we have $\bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right] = (0,2]$

For option (C) Note that for all $n \in \mathbb{N}, 1 - \frac{1}{n} < 1$,

Also, $\lim_{n\to\infty}\left(1 - \frac{1}{n}\right) = 1$

Therefore, $1 \in \bigcup_{n=1}^{\infty}\left(1 - \frac{1}{n}, 2\right]$, But $1 \notin (1,2]$

Hence, $\bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ is arbitrary

For Option (D) Let $x \in [1,2]$ be arbitrary then $1 \leq x \leq 2$. Since $\forall\, n \in \mathbb{N}, 1 - \frac{1}{n} < 1 \leq x$,

$\Rightarrow x \in \left[1 - \frac{1}{n}, 2\right] \forall\, n \in \mathbb{N}$

Since, x was arbitrary, therefore $[1,2] \subseteq \bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ ....(3)

Let $y \in \bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ is arbitrary

Then, $y \in \left[1 - \frac{1}{n}, 2\right] \forall\, n \in \mathbb{N}$

Also, $\lim_{n\to\infty}\left(1 - \frac{1}{n}\right) = 1$, then $y \geq 1$.

So, $y \in [1,2]$

Since y was choose arbitrary, there fore $\bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right] \subseteq [1,2]$ ....(4)

From (3) and (4),

We have, $\bigcap_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right] = [1,2]$

Hence option (2) and (4) are correct.

10. **Explanation:** For option (A) and (C) Let us consider a set $A = (-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$

Clearly, A is open, as union of two open intervals

Also, note that $A \neq R$ as $\sqrt{2} \in \mathbb{R}$ But as $\sqrt{2} \notin A$ and is uncountable set.

Therefore, $Q \subset A \subset \mathbb{R}$, But $A \neq R$ and A is uncountable but not closed.

It contradicts both (A) and (C)

Also, option (D) is wrong, as if we consider $A = \mathbb{Q} \cup \{\sqrt{2}, x\}$ clearly $Q \subset A \subset \mathbb{R}$. Also, A is countable, as union of two countable set. But A is not closed in $\mathbb{R}$

Also, option (2) is correct, as given that $Q \subset A$ then $\overline{Q} \subset \overline{A}$ Then $\overline{Q} \subseteq \overline{A}$. But $\overline{Q} = \mathbb{R}$ and $\overline{A} = A$ (As A is closed)

Then we have $\mathbb{R} \subseteq A$, this given $A = \mathbb{R}$.

Hence option (2) is correct

11. **Explanation:**

For option (A) Since $\mathbb{N}'$ is the countable product of countable sets. Since countable product of countable sets is countable. So, $\mathbb{N}'$ is countable

(C) the set of integers is countable

(D) Since, $\mathbb{Q}$ is countable and $\sqrt{2} \in \mathbb{Q}$ then $\sqrt{2}\mathbb{Q}$ is also countable.

(B) $\{0,1\}^{\mathbb{N}}$, is the set of all sequences which takes the value either 0 and 1.

Suppose that this set is countable then there exist bijection with $\mathbb{N}$.

Let, $\{0,1\}^{\mathbb{N}} = \{x_{1n}, x_{2n}, x_{3n} \ldots.\}$, where each x_{in}, is a sequence and $f(i) = x_{in}$

Let us consider a sequence X, such that

$X_1 \neq X_{11}$(if $X_{11} = 0$, we define $X_1 = 1$ or vice versa)

$X_2 \neq X_{22}$ (same as above)

$\vdots \quad \vdots$

$\vdots \quad \vdots$

$X_n \neq X_{nn}$

Hence, $\langle x_n \rangle$ is a sequence having value either O or 1 but not in the range set of f. Hence $\{0,1\}^{\mathbb{N}}$ is Uncountable

Hence, option (2) is correct.

12. **Explanation:** For example,

Consider, $X = \mathbb{Z}$ and $A = \mathbb{N}$

Clearly, both are countably infinite subset of $\mathbb{R}$, then

$X \backslash A = \{x \in X | x \in A\}$

$= \{x \in \mathbb{Z} | x \notin \mathbb{N}\}$

$= \{0, -1, -2, \ldots\}$ is countable infinite set.

Also $X \backslash A \subset X$, then X cannot be uncountable.

13. **Explanation:** consider the finite series

$\sum u_n = x + \frac{2^2x^2}{\lfloor 2} + \frac{3^3x^3}{\lfloor 3} + \cdots$

Hence, $u_n = \frac{n^n.x^n}{\lfloor 3}$ and $u_{n+1} = \frac{(n+1)^{n+1}x^{n+1}}{\lfloor n+1}$

$\therefore \frac{u_{n+1}}{u_n} = \frac{(n+1)^{n+1}.x^{n+1}.\lfloor n}{\lfloor n+1 .n^n.x^n} = \frac{(n+1)^n}{n^n}.x$

$= \left(1+\frac{1}{n}\right)^n.x$

Therefore, $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n.x = ex$

By D'Alembert ratio test given infinite series is convergent $ex < 1$ i.e. $x < \frac{1}{e}$

Hence option (1) is correct

(Note that because of options, we didn't check the convergence for $ex = 1$, otherwise we need to check.)

14. **Explanation:**

$A = \{x \in \mathbb{Q}: -1 < x < 0\} \cup \mathbb{N}$

$= ((-1,0) \cap \mathbb{Q}) \cup \mathbb{N}$

Clearly, A is unbounded set, as N is unbounded.

Clearly, A is infinite set as both natural number and rational between-1 and 0, are infinite in numbers.

Clearly, N have no limit point, But the set of limit points of $(-1, \mathbb{Q}) \cap \mathbb{Q}$ is the whole interval [-1, 0] in $\mathbb{R}$.

Hence option (2) is correct.

15. **Explanation:** Since, $S = \prod_{n=1}^{\infty}\left[-\frac{1}{n}, 1+\frac{1}{n}\right]$

$= \prod_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

Let $x \in [0,1]$ is arbitrary, then for all $n \in \mathbb{N}$,

we have $\frac{-1}{n} < 0 \le x \le 1 < 1 + \frac{1}{n}$

$\Rightarrow x \in \left[\frac{-1}{n}, 1+\frac{1}{n}\right]$ for all $n \in \mathbb{N}$

Since x was arbitrary. Therefore

$[0,1] \subseteq \cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

Now let $y \in \cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

$\Rightarrow y \in \left[\frac{-1}{n}, 1+\frac{1}{n}\right]$ for all \$n \in \mathbb{N}$

$\Rightarrow \frac{-1}{n} \le y \le 1 + \frac{1}{n} \forall\, n \in \mathbb{N}$

Since, $\lim\limits_{n\to\infty}\left(\frac{-1}{n} = 0\right) = 0$ and $\lim\limits_{n\to\infty}\left(\frac{-1}{n}\right) = 1 + 0 = 1$

Then by squeeze, theorem

$0 \le y \le 1$ i.e. $y \in [0,1]$

Since y was arbitrary, we have

$\cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right] \subseteq [0,1]$

From (1) and (2), we have

$\cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right] = [0,1]$

Hence option (1) is correct

16. **Explanation:** Given that

$S = \left\{\frac{1}{n}: n \in \mathbb{N} \text{ and } n \text{ is prime}\right\}$

$= \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$

Carty, $p \ge 2$ for every prime $p \in \mathbb{N}$.

$\therefore \frac{1}{P} \le \frac{1}{2}$ for every prime $p \in \mathbb{N}$.

$\Rightarrow Sup(S) = \frac{1}{2}$

$\left(\text{as} \frac{1}{2} \text{ is an upper bound of S and element of S}\right)$

Also, $T = \{x^2 : x \in \mathbb{R}\}$

Clearly, $x^2 \ge 0 \forall\, x \in \mathbb{R}$

Hence, is a lower bound of T and $0 \in T(\text{as } x = 0 \in \mathbb{R})$, therefore $\inf T = 0$

Here option (3) is correct.

17. **Explanation:**

(I) Let, us define $f: N \to I$ as $f(n) = \frac{1}{n}$ clearly f is one-one correspondence.

(II) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of all integers and integers are countable hence there must exist a one-one correspondence with natural number.

(III) $\left\{\frac{P}{Q}: p, q \in \mathbb{N}\right\}$ is the set of all Rational numbers, and set of rational number is countable hence there must exist a one-one correspondence with natural number.

(IV) Clearly, $\left\{\frac{P}{Q}: p, q \in \mathbb{N}\right\}$ is a subset of $\mathbb{Q}$

Since, $\mathbb{Q}$ is countable, so being a subset of countable set, (IV) is also countable.

Since it is infinite set, so there must exist a one-one correspondence with natural number.

Hence option (4) is correct.

18. **Explanation:** $X = \{(x,y) \in R^2; x \in \mathbb{Q}\; y \in \mathbb{R}\backslash\mathbb{Q}\}$

Now we have to check whether X is open in $\mathbb{R}^2$ or dense in $\mathbb{R}^2$.

Open: $(\alpha, \beta) \in X$ be arbitrary point, then every neighbourhood of (α, β) in $\mathbb{R}^2$ contain a point where $\alpha \notin \mathbb{Q}$ and $\beta \notin \mathbb{Q}^c$ so (α, β) need not be interior point of $X, so\ X$ is not open

Dense: Let $(a,b) \in \mathbb{R}^2$ Then every neighbourhood of (a,b) in $\mathbb{R}^2$ contains point where $a \in \mathbb{Q}, b \in \mathbb{Q}^2$ and $a \in \mathbb{Q}^c, b \in \mathbb{Q}$

$i.e.$ neighbourhood of $(a,b) \in X$ and $(a,b) \in X^c$

$i.e$ (a,b) is a boundary point of $\mathbb{R}^2$

So, $\overline{X} = \mathbb{R}^2$,therefore X is dense in $\mathbb{R}^2$

Hence option, (3) is correct.

19. **Explanation:** Given that,

$S = \{x \in \mathbb{R}: 3 - x^2 > 0\}$

$= \{x \in \mathbb{R}: 3 > x^2\}$

$= \{x \in \mathbb{R}: |x| < \sqrt{3}\}$

$= (-\sqrt{3}, \sqrt{3})$

Clearly, S is bounded in $\mathbb{R}$, also $\sup S = \sqrt{3} \notin \mathbb{Q}$

Hence option (3) is correct.

20. **Explanation:** Note that any polynomial is of the form

$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots$ where each $a_1 \in \mathbb{Q}$

Since rational number is countable and countable product of countable sets is always countable.

So the set P is countable.

Also S the set of all sequences of natural number is uncountable set

Let A be the set of all sequences of 0 and 1. Then by Using cantor's diagonal argument this set is uncountable. So S being superset of this set is always uncountable.

Hence option (4) is correct

21. **Explanation:**

$S = \cap_{n=1}^{\infty} \left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right]$

Since, $\lim\limits_{n\to\infty} \frac{1}{n} = 0$,, then $\lim\limits_{n\to\infty} \left(2 - \frac{1}{n}\right) = 2$, and $2 - \frac{1}{n} < 2 \forall n$

Also, $\lim\limits_{n\to\infty} \frac{1}{n} = 0$, then $\lim\limits_{n\to\infty} \left(3 + \frac{1}{n}\right) = 3$ and $3 + \frac{1}{n} > 3 \forall n$

$\therefore S = \cap_{n=1}^{\infty} \left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right] = [2,3]$

Hence option (2) is correct

22. **Explanation:** Note that $\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 1\ ;\ n = 4m+1 \\ 0\ ;\ n = 4m+2 \\ -1\ ;\ n = 4m+1 \\ 0\ ;\ n = 4m \end{cases}$

for same $m \in \mathbb{Z}$

Therefore we have three subsequence as,

$X_n = 1 \forall n = 2m,\ \ m \in \mathbb{Z}$

$X_n = n \, \forall \, n = 4m + 1,\ \ m \in \mathbb{Z}$

$X_n = \frac{1}{n} \forall n = 4m + 3,\ \ m \in \mathbb{Z}$

Since, $\lim\limits_{n\to\infty} X_{4n+1} = \lim\limits_{n\to\infty} (4m + 1) = +\infty$

$\lim\limits_{n\to\infty} X_{4n+1} = \lim\limits_{n\to\infty} (4m + 1) = 0$

$\therefore \lim \sup a_n = +\infty \ and \lim \inf a_n = 0$

Hence option (2) is correct.

23. **Explanation:** Note that $\frac{1}{n} > 0 \, \forall \, n$ and $\lim\limits_{n\to\infty} \frac{1}{n} = 0$

$\therefore \cap_{n=1}^{\infty} \left[0, \frac{1}{n}\right] = \{0\} = S$

Hence $\sup S = \inf S = 0$

Hence option (4) is correct

24. **Explanation:** Given : $x^2 \leq 5$

$\Rightarrow x^2 - 5 \leq 0$

$\Rightarrow (x - \sqrt{5})(x + \sqrt{5}) = 0$

i.e. $-\sqrt{5} \leq x \leq \sqrt{5}$

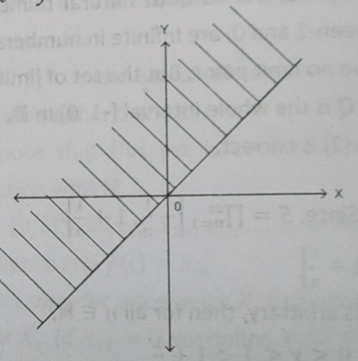
$\Rightarrow x \in [-\sqrt{5}, \sqrt{5}]$

$\Rightarrow$ S is closed interval

$\Rightarrow$ option (3) is correct

25. **Explanation:** Given: $x < y$

$\Rightarrow x - y < 0$



Which is neither R nor set of all negative real numbers

$\Rightarrow$ all options are wrong

26. **Explanation:** Given: $X \subset \mathbb{N}$ is a non-empty finite set and $Y \subset \mathbb{N}$ is an infinite set

Here, $A = \{x - y : x \in X \ and \ y \in Y\}$

Let $X = \{1,2,3,4,5\}$

And $Y = 2\mathbb{N}$

$A = x - y = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

$= \mathbb{Z}$

$\therefore \inf(A) = -\infty$ and $\sup(A) < \infty$

$\Rightarrow$ option (2) is correct

27. **Explanation:**

$A, B \subset X$

Let $X = \mathbb{Z}$

And $A = B = \mathbb{N}$

Now, $(A \cup B)\setminus(A \cap B) = (\mathbb{N} \cup \mathbb{N})\setminus(\mathbb{N} \cap \mathbb{N})$

$= \mathbb{N}\setminus\mathbb{N}$

$= \phi$

⇒ finite set

⇒ option (1), (2) and (3) are wrong and option (4) is correct

28. Explanation: Since total number of subsets of

$X = 2^{2m+1}$

$\therefore 2^{2m+1} = {}^{2m+1}C_0 + {}^{2m+1}C_1 + \cdots + {}^{2m+1}C_m + {}^{2m+1}C_{m+1} + {}^{2m+1}C_{m+2} + \cdots + {}^{2m+1}C_{2m+1}$

Since ${}^nC_r = {}^nC_{n-r}$

∴ The subsets having $m+1$ elements

$2^{2m+1} =$

$2\underbrace{({}^{2m+1}C_{m+1} + {}^{2m+1}C_{m+2} + \cdots + {}^{2m+1}C_{2m+1})}_{\alpha}$

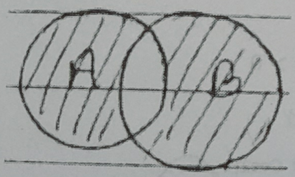
$2^{m+1} = 2\alpha$

$\therefore \alpha = \frac{2^{2m+1}}{2} = 2^{2m+1-1} = 2^{2m}$

⇒ option (3) is correct

29. Explanation:

Given: $A\Delta B = (A\setminus B) \cup (B\setminus A)$

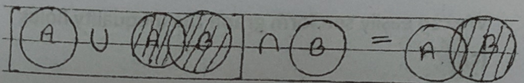


(A) $A \cap (B\Delta A) = A \cap [(A\setminus B) \cup (B\setminus A)]$

$\neq \phi$

⇒ option (A) is wrong

(B) $\big(A \cup (A \,\Delta\, B)\big) \cap B = B$



⇒ option (B) is wrong

(c) $A \,\Delta\, \mathrm{B} = (A \cup B) \cap (A^c \cap B^c)$

$= (A \cup B) \cap (A \cap B)^c$

⇒ option (C) is wrong

⇒ option (4) is correct

30. Explanation:

a) Let $\mathrm{F} : [0,1] \to \mathrm{S} = \mathrm{S}_1 \cup \mathrm{S}_2$

here [0, 1] is uncountable set and $\mathrm{S} = \mathrm{S}_1 \cup \mathrm{S}_2$ which is countable.

∴ There does not exist a map from uncountable set to countable set

⇒ option (a) and (b) are wrong

Here $\mathrm{S}_1 = \{t_1 + t_2 \,|t_1 \in \mathrm{Q}_n[0,1], t_2 = \sqrt{2}\}$

And $\mathrm{S}_2 = \{t_1 + t_2 | t_1 \in \mathrm{Q}_n[0,1], t_2 = \sqrt{3}\}$

Both are countable

If F is one-one → every element of [0,1] goes to different element of S

c) Define $\mathrm{F} : \mathrm{S}_1 \to \mathrm{S}_2$ s.t

$$\mathrm{F}(x) = \begin{cases} x, & \forall\ x \in \mathrm{Q}_n[0,1] \\ \sqrt{3}, & x = \sqrt{2} \end{cases}$$

⇒ F is bijective

31. Explanation:

Given $\mathrm{F} : \mathbb{R} \to \mathbb{R}$ is a continuous map and $\mathrm{S} = \left\{\pi + \frac{(-1)^n}{2n+1} \,|\, n \in \mathbb{N}\right\} \cup (0,1)$

⇒ S is bounded

⇒ $\bar{\mathrm{S}}$ is closed and bounded

⇒ $\bar{\mathrm{S}}$ is compact and bounded

And under continuous map, compact set goes to compact set

⇒ $\mathrm{F}(\bar{\mathrm{S}})$ is compact

And $\mathrm{F} : (0,1) \to \mathbb{R}$ S.t $\mathrm{F}(x) = 0$ and singleton set is closed

⇒ Its image is also closed

32. Explanation:

We know that, A subset of $\mathbb{R}$ is compact iff it is closed and bounded

A) WkT, $\left\{\frac{1}{n}\right\}$ is bdd and {0} is closed as every singleton set is closed and $[0, \pi]$ is closed

⇒ (A) is compact

C) {1,2,3} and [4, 5] are closed set but $6 \notin \left\{6 + \frac{1}{n} : n \in \mathbb{N}\right\}$

⇒ It is not closed

⇒ It is not compact

B) as $0 \notin [0, 1]$ it is not closed and $0 \notin \left\{-\frac{1}{n}; n \in \mathbb{N}\right\}$

∴ The set is not closed

⇒ It is not compact

D) $1 \notin (1,2]$ and $1 \notin \left\{1 - \frac{1}{n}; n \in \mathbb{N}\right\}$

∴ It is not closed

⇒ It is not compact

33. Explanation:

$|\mathbb{R}| = z^x = c$

(A) Set of all real sequence

$S = \{f \mid f: N \to \mathbb{R}\}$

$|S| = C^{x0} = C$

A correct

(B) Set of all continuous function $f : \mathbb{R} \to \mathbb{R}\, S = \{P \mid f : \mathbb{R} \to \mathbb{R}\ continuous\ functions\}$

$|S| = C$

B correct

(C) $5 = \{f \mid f: \mathbb{R} - [0,1]\}$

$|s| = 2^C > C$

C incorrect

34. Explanation:

$f : \mathbb{R}^3 \to \mathbb{R}$ be continuous function such that

$f(\propto v) = f(v)\ \forall x \in \mathbb{R}\ x > 0\ \forall \in \mathbb{R}$

(1) $\mathbb{R}^n$ is compact

Image of compact space under continuous map is compact

$\Rightarrow f(\mathbb{R}^3) \subseteq \mathbb{R}$ is compact

So statement A is true.

(2) let

$f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$f(x, y, z) = C \in \mathbb{R}$

Then

$f(\mathbb{R}^3) = \{C\}$

Which is countable

$\Rightarrow$ statement (2) is incorrect

35. Explanation:

Let

$C = Q\ D = \mathbb{N}$

$C\mathbb{D} = Q - \mathbb{N}$

$(C\mathbb{D})^0 = \phi$

36. Explanation:

$S = \{(x, y) \in \mathbb{R}^2 \mid y = x\ or\ |x| + |y| = 1\}$

(1)

$(x, x) \in S$

$\because y = x$

$\therefore S$ is reflexive

(B) $L(p, q) \in S, (at) \in S$

$\Rightarrow$ either

$P = q\ or\ |p| + |q| = 1$ & either $q = b + |y|(pq) = 1$

(1)

$P = q = x \Rightarrow (P\,r) \in S$

(2)

$|P| + |q| = 1$

$|r| + |q| - 1$

$|a| + |b| = 1$

$|r| + 1 - |t| = 1$

$|q| = 1 - |t|$

$|r| + |t| = 0$

$|r| = |t|$

$\Rightarrow S$ is transitive

37. Explanation:

$S = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\} \subset \mathbb{R}$

(1)

$f : Q \times Q \to \mathbb{R}$

$g = \mathbb{R} \to S$

g of (x) :

g of $(x) : Q \times Q \to S.$

To become surjective 'g' must be surjective

& $g : \mathbb{R} \to S$

$|\mathbb{R}| > |S|$

$\Rightarrow g$ is surjective

g of (x) can be surjective

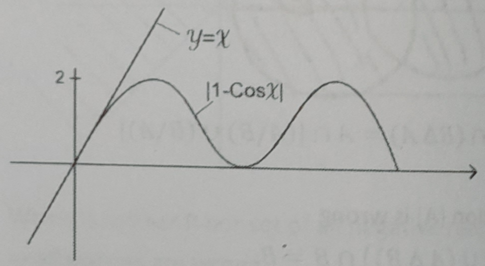
So option (1) is incorrect

(2)

$|S| < \left|\frac{R^2}{\mathbb{Z}}\right|$

$\Rightarrow \nexists$ any onto map.

(B) $|1 - \cos x| \le x\ x > 0$



We can easily see form graph the inequality holds

(c)

$|\sin x - \sin y| \le |x - y|$

Since $|\sin x| \le x$

So all statements are correct

38. Explanation:

(1) Arbitrary union of open sets is open

So statement (A) true.

(2)

Let

$\left(-\frac{1}{n}, \frac{1}{n}\right)$

$\cap \left(-\frac{1}{n}, \frac{1}{n}\right) = \{0\}$

So A is true B is incorrect

39. Explanation:

$(a, b) \in \mathbb{R}$ if $a - b$ is multiple of 5 or 6

i.e. $\frac{4}{a-b}$ or $\frac{c}{a-b}$

(1) Let $(a, b) \in \mathbb{R}$

$\Rightarrow 4|a - b \; or \; 6|a - b$

$\Rightarrow 4|(b - a) \; or \; 6| - (b - a)$

$\Rightarrow R$ is symmetric

(A) is correct

(2) Let $(a, b) \in \mathbb{R} \; (b, c) \in \mathbb{R}$

$\Rightarrow 4|a - b \; or \; 6| \; a - b \; \& \; 4|b - c \; or \; 6|b - c$

$\Rightarrow ab = 4k,$ or $a - b = 6k_2 \; \& \; b - c = 4k_3$ or $b - c = 6k_4$

$a - 4k_3 - c = 4k_1$ or $a - 6k_4 - c = 6k_2 \Rightarrow b = 4(3 + c \; or \; b = 6k_a + C)$

$a - c = 4(k_1 + k_4)$ or $a - c = 6(k_4 + k_2)$

$\Rightarrow 4 \mid a - c$ or $6 \mid a - c$

$\Rightarrow R$ is transitive

So 2 statement are correct

Unit 1.2: Sequences of Real Numbers

[CUET PG]

[CUET PG 2022]

1. The value of $\sqrt[n]{n}$ is
   (1) 0 (2) ∞
   (3) 1 (4) 2

[CUET PG 2022]

2. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

   Assertion A:

   The sequence $\left\{(-1)^n, \frac{n+1}{n}\right\}$ is not convergent

   Reason R:

   Every monotonic sequence is convergent

   In the light of the above statements, choose the most appropriate answer from the options given below
   (1) Both A and R are correct and R is the correct explanation of A
   (2) Both A and R are correct but R is not the correct explanation of A
   (3) A is not correct but R is correct
   (4) A is correct but R is not correct

[CUET PG 2023]

3. The positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if
   (1) $p > 0$ (2) $p > 1$
   (3) $p < 1$ (4) $p \geq 1$

[CUET PG 2023]

4. The value of $\left[\frac{2}{1}\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \cdots \left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}}$ is
   (1) 1 (2) 0
   (3) e (4) $\frac{1}{e}$

[CUET PG 2023]

5. The sequence $\langle\frac{(-1)^n}{n}\rangle$ is
   (1) oscillatory sequence
   (2) divergent sequence
   (3) either Cauchy or convergent sequence
   (4) both Cauchy and convergent

Answer Key				
1	2	3	4	5
3	3	2	2	2

[CU-CET]

[CUCET 2016]

1. Let p, q be two reals such that $p > q > 0$. Define the sequence $\{x_n\}$, where $x_1 = p + q$ and $x_n = x_1 - \frac{pq}{x_{n-1}}$ for $n \geq 2$. Then for all n, x_n is equal to one of the following and determine it.
   (1) $x_n = \frac{p^{n+1}-q^{n+1}}{p^n-q^n}$ (2) $x_n = \frac{p^{n+1}+q^{n+1}}{p^n+q^n}$
   (3) $x_n = \frac{(pq)^n}{p^n+q^n}$ (4) $x_n = \frac{(pq)^n}{p^n-q^n}$

[CUCET 2016]

2. Which one of the following statements is wrong?
   (1) Every convergent sequence of reals is necessarily bounded
   (2) Every sequence of reals has a monotone subsequence
   (3) Every monotone increasing sequence which is bounded above is convergent
   (4) Every sequence which is bounded above has a convergent subsequence

[CUCET 2016]

3. The sequence $\left\{\frac{n}{2^n} + 1\right\}$
   (1) Is bounded but not convergent
   (2) Is convergent and converges to 0
   (3) Is convergent and converges to 1
   (4) Is monotone increasing

[CUCET 2016]

4. The minimum value of the sum $\sum_{k=1}^{n} n_k^2$ of reals satisfying $\sum_{k=1}^{n} a_k = 1$ is
   (1) $\frac{1}{\sqrt{n}}$ (2) $\frac{1}{n}$
   (3) $\frac{1}{n^2}$ (4) $\frac{1}{n^3}$

[CUCET 2016]

5. Consider the sequences $\{a_n\}$ and $\{b_n\}$ where $a_n = \left(1+\frac{1}{n}\right)^n$ and $b_n = \left(1+\frac{1}{n}\right)^{n+1}$ for all $n \in N$. Then.
   (1) both sequences are monotone increasing
   (2) both sequences are monotone decreasing
   (3) one of these two sequences is monotone increasing and the other one is monotone decreasing
   (4) both the sequences are unbounded

[CUCET 2017]

6. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers defined as $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = a_n + (-1)^n 2^{-n}$, $b_n = \frac{2a_{n+1}-a_n}{a_n}$. Then

(1) $\{a_n\}$ converges to zero and $\{b_n\}$ is a Cauchy sequence

(2) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is a Cauchy sequence

(3) $\{a_n\}$ converges to zero and $\{b_n\}$ is not a convergent sequence

(4) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is not a convergent sequence

[CUCET 2017]

7. The sequence $\{(-1)^n\}_{n=1}^{\infty}$ is

(1) bounded and convergent

(2) convergent and unbounded

(3) bounded and divergent

(4) divergent and unbounded

[CUCET 2017]

8. If sequences of real numbers $\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are such that, $b_n = a_{2n}$ and $c_n = a_{2n+1}$, then $\{a_n\}_{n=1}^{\infty}$ is convergent implies

(1) $\{b_n\}_{n=1}^{\infty}$ is convergent but $\{c_n\}_{n=1}^{\infty}$ need not be convergent

(2) $\{c_n\}_{n=1}^{\infty}$ is convergent but $\{b_n\}_{n=1}^{\infty}$ need not be convergent

(3) both $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are convergent

(4) both $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are divergent

[CUCET 2017]

9. $(2^n + 3^n)^{\frac{1}{n}}$ is equal to

(1) 2 (2) 3

(3) 5 (4) 6

[CUCET 2017]

10. Which amongst the following statements is not true?

(1) A sequence cannot converge to more than one limit,

(2) Every convergent sequence is bounded

(3) Every bounded sequence is convergent

(4) Limit of a convergent sequence is unique

[CUCET 2017]

11. The sequence $a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}$

(1) converges to 0

(2) converges to $\frac{1}{2}$

(3) converges to $\frac{1}{4}$

(4) does not converge

[CUCET2017]

12. Let $a_n = sin\ sin\ \frac{1}{n^2}, n = 1,2,\ldots$, then

(1) $a_n = 1$

(2) $\sum_{n=1}^{\infty} a_n$ converges

(3) $sup\ sup\ a_n \neq inf\ a_n$

(4) $\sum_{n=1}^{\infty} a_n$ diverges

[CUCET 2019]

13. The sequence $\left\{\frac{1}{n}\right\}$ is

(1) convergent (2) divergent

(3) oscillatory (4) unbounded

[CUCET 2019]

14. $\frac{2n-3}{n+1}$ equals

(1) 0 (2) 1

(3) 2 (4) e

[CUCET 2019]

15. $\left(1 + \frac{1}{n}\right)^n$ equals

(1) e (2) $\frac{1}{e}$

(3) 0 (4) 1

[CUCET 2019]

16. Which of the following statements is false?

(1) Every bounded sequence is convergent.

(2) Every convergent sequence is bounded.

(3) Every bounded sequence has a limit point.

(4) Every convergent sequence has a unique limit.

[CUCET 2019]

17. The upper limit of the sequence $\{(-1)^n\}$ is

(1) 1 (2)-1

(3) 0 (4) 2

[CUCET 2020]

18. The sequence $< e^n / n >$ is

(1) a monotonically increasing sequence

(2) a monotonically decreasing sequence

(3) bounded above but not bounded below

(4) a bounded sequence

[CUCET 2020]

19. The sequence $< x_n >$, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

(1) satisfies the Cauchy's criterion

(2) does not satisfy the Cauchy's criterion

(3) is a monotonically decreasing sequence

(4) possesses the limit 0

[CUCET 2020]

20. Let $x_n = 2^{2n}(1\text{-}\cos\frac{1}{2^n})$ then the sequence x_n

(1) does not converge

(2) converges to 0

(3) converges to $\frac{1}{2}$

(4) converges to $\frac{1}{4}$

[CUCET 2021]

21. Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence of real numbers. Then
    (1) There is a subsequence of $\{a_n\}_{n=1}^{\infty}$ which is convergent
    (2) Every subsequence of $\{a_n\}_{n=1}^{\infty}$ is convergent
    (3) There is exactly one subsequence of $\{a_n\}_{n=1}^{\infty}$ which is convergent
    (4) None of these

[CUCET 2021]

22. Which one of the following is incorrect?
    (1) Every subsequence of a convergent sequence of real numbers is convergent
    (2) Every convergent sequence of real numbers is bounded
    (3) Every bounded infinite set of real numbers has at least one limit point
    (4) Every bounded sequence of real numbers is convergent

Answer Key

1	2	3	4	5	6	7	8	9	10
1	4	3	2	3	2	3	3	2	3
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	2	1	3	1	1	1	1	2	3
21	**22**								
1	4								

[PRACTICE QUESTIONS]

1. $\overline{\lim}$ and $\underline{\lim}$ of the sequence $\cos\frac{n\pi}{4} + \sin\frac{n\pi}{4}$ are respectively
    (1) $\sqrt{2}, -\sqrt{2}$
    (2) $\sqrt{2}, -1$
    (3) $1, -\sqrt{2}$
    (4) $1, -1$

2. If a sequence $\{a_n\}_{n=1}^{\infty}$ of elements in the interval $(-1, 1)$ is given, then which one of the following is true?
    (1) Every limit point of $\{a_n\}$ is in $(-1,1)$.
    (2) Every limit point of $\{a_n\}$ is in $[-1,1]$.
    (3) The limit points of $\{a_n\}$ can only be in $\{-1,0,1\}$.
    (4) the limit point of $\{a_n\}$ cannot be in $\{-1,0,1\}$.

3. The value of $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right]$ is
    (1) 1
    (2) 0
    (3) ∞
    (4) $\frac{1}{2}$

4. The value of $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{\frac{1}{n}}$ is
    (1) 1
    (2) e
    (3) 0
    (4) $\sqrt[n]{e}$

5. For $a_1 > 0$, the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_{n+1} = 1 + \frac{1}{a_n}$ $\forall n \geq 1$,converges to
    (1) $\frac{\sqrt{5}}{2}$
    (2) $\frac{\sqrt{5}+1}{2}$
    (3) $\frac{\sqrt{5}-1}{2}$
    (4) $\frac{1}{\sqrt{5}}$

6. If $f(x) = \begin{cases} \frac{1}{a^{n-1}}, \frac{1}{a^n}, x \leq \frac{1}{a^{n-1}}, n = 1,2,3,\ldots, \text{ and } a > 1 \\ 0. x = 0 \end{cases}$ then
    (1) f is integrable on $[0,1]$ and $\int_0^1 f\, dx = a$
    (2) f is integrable on $[0,1]$ and $\int_0^1 f\, dx = \frac{a}{a+1}$
    (3) f is not integrable on $[0,1]$
    (4) f is integrable on $[0,1]$ and $\int_0^1 f\, dx = \frac{a+1}{a}$

7. If $\{a_n\}_{n=1}^{\infty}$ and $\{b\}_{n=1}^{\infty}$, be sequences of real numbers such that $\lim_{n\to\infty} a_n - I$ and $\lim_{n\to\infty} b_n = m$ (where $I, m \in \mathbb{R}$), then $\lim_{n\to\infty}\frac{1}{n}(a_1b_n + a_2b_{n-1} + \cdots + a_nb_1)$ is:
    (1) l^2m
    (2) lm
    (3) lm
    (4) $lm - (l+m)$

8. If $\lim_{n\to\infty}|x_n| = |l|$ (where $I \in \mathbb{R}$, then the sequence $\{x_n\}_{n-1}^{\infty}$:
    (1) converges to I
    (2) converges either to /or to $-I$
    (3) may not be convergent
    (4) is convergent, but not to I

9. Which of the following statements is true?
    (1) If x_0 is a limit point of a sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers, then there exists a subsequence of $\{x_{nk}\}_{k=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$, which converges to x_0
    (2) Every monotonic increasing sequence of real numbers is convergent.
    (3) Every monotonic decreasing sequence of positive real numbers is divergent.
    (4) Every sequence of real numbers has a convergent subsequence.

10. Let the sequence $\{x_n\}_{n=1}^{\infty}$ be defined as $x_1 = 1$ and $x_{n+1} = 1 + \sqrt{x_n}$ for $n \geq 1$. Then:
(1) sequence $\{x_n\}_{n=1}^{\infty}$ is monotonically increasing
(2) sequence $\{x_n\}_{n=1}^{\infty}$ is monotonically decreasing
(3) sequence $\{x_n\}_{n=1}^{\infty}$ is not convergent
(4) sequence $\{x_n\}_{n=1}^{\infty}$ is not bounded

11. Let the sequence $\{x_n\}_{n=1}^{\infty}$ be defined as $x_1 = \frac{1}{3}, x_{2n} = \frac{1}{2}x_{2n-1}$ and $x_{2n+1} = \frac{1}{3} + x_{2n}$ for $n = 1,2,3\ldots$ Then :
(1) $\limsup\{x_n\} = \frac{1}{2}$ and $\liminf\{x_n\} = \frac{1}{3}$
(2) $\limsup\{x_n\} = \frac{1}{2}$ and $\liminf\{x_n\} = \frac{1}{3^2}$
(3) $\limsup\{x_n\} = \frac{1}{3}$ and $\liminf\{x_n\} = \frac{1}{2}$
(4) $\limsup\{x_n\} = \frac{1}{2}$ and $\liminf\{x_n\} = \frac{1}{6}$

12. Which of the following sequences is not convergent?
(1) $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$
(2) $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$
(3) $\left\{1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right\}_{n=1}^{\infty}$
(4) $\left\{\frac{2^n}{n^n}\right\}_{n=1}^{\infty}$

13. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of real numbers and α be a non-zero real number such that $\lim\limits_{n\to\infty}(\alpha_1 + \alpha_2 + \cdots + \alpha_n) = \alpha$ then $\lim\limits_{n\to\infty}(a_n)$ is :
(1) α (2) $\alpha + 1$
(3) $\alpha + 2$ (4) 0

14. Let $a_n = 2^n + n^2$ for $n \leq 100$ and $a_n = 3 + (-1)^n \frac{n^{\wedge}}{2^n}$ for n > 100. Then
(1) $\langle a_n \rangle$ is a Cauchy sequence
(2) $\langle a_n \rangle$ is an unbounded sequence
(3) $\langle a_n \rangle$ has exactly three limit points
(4) $\langle a_n \rangle$ has two convergent subsequences converging to two different points.

15. Which of the following statements about a sequence of real number is not true?
(1) Every bounded sequence has a convergent subsequence
(2) Every sequence has a monotonic subsequence
(3) Every sequence has a limit point
(4) Every sequence has a countable number of terms

16. $\langle a_n \rangle = \langle 1,1,1/2,1,1/2,1/3,1/2,1/3,1/4 \rangle$ be a sequence of real numbers. Then which of the following is not true?
(1) $\langle a_n \rangle$ has infinite number of limit points
(2) $\limsup_{n\to\alpha} a_n = 1$
(3) $\liminf_{n\to\alpha} a_n = 0$
(4) $\langle a_n \rangle$ has infinite number of convergent subsequences

17. Let $F_n = [-1/n, 1/n]$, for each $n \in N$ and let $F = \cap_{n=1}^{\infty} F_n$
Then which of the following is not true?
(1) F contains finite number of points
(2) $\sup\{|x-y|: x, y \in F\} = 0$
(3) $\inf\{|x-y|: x, y \in F\} = 0$
(4) F is a open set.

18. The sequence $\langle \frac{2^{n+1}+3^{n+1}}{2^n+3^n} \rangle$
(1) 1 (2) 2
(3) 3 (4) 5.

19. The limit of the sequence $\langle \sqrt{(n+1)(n+2)} - n \rangle$ as $n \to \infty$ is
(1) $\sqrt{2} - 1$ (2) 3
(3) 3/2 (4) 0.

20. If $x_n = 1 + (-1)^n + \frac{1}{2^n}$, then
(1) $\limsup x_n = 1$
(2) $\liminf x_n = 1$
(3) x_n is a convergent sequence
(4) $\limsup x_n \neq \liminf x_n$

21. Let $\langle x_n \rangle$ be the sequence defined by $x_1 = 2$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$. then
(1) $\langle x_n \rangle$ converges to rational number
(2) $\langle x_n \rangle$ is an increasing sequence
(3) $\langle x_n \rangle$ converges to $2\sqrt{2}$
(4) $\langle x_n \rangle$ is an decreasing sequence

22. Let $\langle x_n \rangle$ be an unbounded sequence in $\mathbb{R}$. Then
(1) $\langle x_n \rangle$ has a convergent subsequence
(2) $\langle x_n \rangle$ has a subsequence $\langle x_{n_k} \rangle$ such that $x_{n_k} \to 0$
(3) $\langle x_n \rangle$ has a subsequence $\langle x_{n_k} \rangle$ such that, $\frac{1}{x_{n_k}} \to 0$
(4) Every subsequence of $\langle x_n \rangle$ is unbounded

23. If $\{a_n\}$ is a bounded sequence of real numbers, then
(1) $\inf_{n} a_n < \lim_{n\to\infty} \inf a_n$ and $\sup_{n} a_n \le \lim_{n\to\infty} \sup a_n$
(2) $\lim_{n\to\infty} \inf a_n \le \inf_{n} a_n$
(3) $\lim_{n\to\infty} \inf a_n \le \inf_{n} a_n$ and $\sup_{n} a_n \le \lim_{n\to\infty} \sup a_n$
(4) $\inf_{n} a_n \le \lim_{n\to\infty} \inf a_n$ and $\lim_{n\to\infty} \sup a_n \le \sup_{n} a_n$

24. The sequence $\left(n^{1/n}\right)$ is
(1) monotonically decreasing
(2) monotonically increasing
(3) convergent and converges to zero
(4) neither monotonically increasing nor monotonically decreasing

25. Let (x_n) be a sequence defined by:
$x_1 = 3$ and $x_{n+1} = \frac{1}{4-x_n}$
Then
(1) (x_n) is a monotonically decreasing sequence that is not bounded below
(2) (x_n) converges to $2+\sqrt{3}$
(3) (x_n) converges to $2-\sqrt{3}$
(4) (x_n) diverges

26. The limit inferior of the sequence (x_n) where $x - n = 1 + (-1)^n + \frac{1}{3^n}$ is
(1) I
(2) 3
(3) 2
(4) 4

27. If $\langle x_n \rangle$ is a sequence such that $x_n \ge 0$ for every & $n \in N$ and if $\lim_{n\to\infty}((-1)^n x_n)$ exists then which one of the following site?
(1) The sequence $\langle x_n \rangle$ is a Cauchy sequence
(2) The sequence $\langle x_n \rangle$ is not a Cauchy sequence
(3) The sequence $\langle x_n \rangle$ is unbounded.
(4) The sequence $\langle x_n \rangle$ is divergent.

28. If $\langle x_n \rangle$ is a sequence defined as
$x_n = \left[\frac{5+n}{2n}\right]$, for every $n \in \mathbb{D}$
Where [.] denotes the greatest integer faction then $\lim_{n\to\infty} x_n$
(1) 1
(2) $\frac{1}{2}$
(3) does not exist
(4) 0

29. The sequence of real-valued functions
(1) pointwise convergent but not uniformly convergent
(2) only convergent
(3) bounded but not point wise convergent
(4) not bounded

30. Let $\{p_n\}$ be a strictly increasing sequence of prime numbers and let
$x_n = (-1)^{p_n+1}\left(1+\frac{1}{P_n}\right)$ *then*
(1) $\lim_{n\to\infty} x_n = -\frac{1}{2}$
(2) $\lim_{n\to\infty} x_n = -1$
(3) $\lim_{n\to\infty} x_n = 1$
(4) $\lim_{n\to\infty} x_n =$ does not exist.

31. If m and M are respectively the greatest lower bound and the least upper bound of the set $S = \left\{\frac{2x+3}{x+2}, x \ge 0\right\}$ then
(1) $m \in S, M \notin S$
(2) $m \notin S, M \notin S.$
(3) $m \notin S, M \in S.$
(4) $m \in S, M \in S.$

32. Let f be a function from $\mathbb{R} \to \mathbb{R}$. consider the statements
P:There exists M *in* $\mathbb{R}$ such that $|f(x)| \le M$ for all x *in* $\mathbb{R}$ which of the
following statements are equivationt to P
(1) The range of f *is a bounded set of lR*
(2) $|f|$ *is a bounded Function.*
(3) f *is taking all values between* $-M$ *and* M.
(4) $|f|$*is taking all values between* 0 *and* $M/2$.

33. We say that a sequence (a_n) does NOT converge to l if
(1) $\forall_{\in} > 0, \forall n_0 \in \mathbb{N}, \forall n \ge n_0$ *we have* $|a_n - 1| > \in$
(2) $\forall_{\in} > 0, \forall n_0 \in \mathbb{N}, \exists n \ge n_0$ *such that* $|a_n - 1| > \in$
(3) $\exists_{\in} > 0, \forall n_0 \in \mathbb{N}, \exists n \ge n_0$ *such that* $|a_n - 1| > \in$
(4) $\exists_{\in} > 0, \forall n_0 \in \mathbb{N}, \forall n \ge n_0$ *we have* $|a_n - 1| > \in$

34. Consider a sequence $(a_{,n})$ of positive numbers satisfying the condition
$a_n a_{n+2} \le a_{n+1}^2 \; \forall n \in \mathbb{N}$ *then* (a_n)*is a*
(1) convergent sequence if $a_1 \ne 2u_2.$
(2) monotonir:allv increasing sequence if $a_1 \ne 2a_2$
(3) convergent sequence if $a_1 = 2a_2$
(4) monotonically increasing seqrrence if $a_1 = 2a_2$

35. Consider a sequence (a_n) of real numbers. Which of the following condition imply that (a_n) is convergent?

(1) $|a_{n+1} - a_n| < \frac{1}{2}, \forall n \in \mathbb{N}$.

(2) $|a_{n+1} - a_n| < \frac{1}{3^n}, \forall n \in \mathbb{N}$.

(3) $a_n > 0, \forall n \in \mathbb{N}$ *and* a_n is monotonically increasing

(4) $a_n > 0, \forall n \in \mathbb{N}$ and a_n is monotonically decreasing

36. Consider the following two statements.

S_1: If (a_n) is any real sequence, then $\left(\frac{a_n}{1+|a_n|}\right)$ has a convergent subsequence.

S_2: If every subsequence of (a_n)has a convergent subsequence, then $(a_n)$is bounded.

Which of the following statements is true?

(1) Both S_1 and S_2 are true.

(2) Both S_1 *and* S_2 *are false.*

(3) S_1 is false but S_2 is true.

(4) S_1 is true but S_2 is false.

37. Let $a_n=(-1)^{n+1}\left(1+\frac{1}{2n+1}\right)$ *for* $n \geq 1$. which of the following is correct?

(1) *lim* sup a_n=1 and *lim* in a_n=-1.

(2) *lim* sup a_n=1 and *lim* in a_n=1.

(3) *lim* sup a_n=-1 and *lim* in a_n=-1.

(4) *lim* sup a_n=1 and *lim* in a_n=1.

38. Let $\{X_n\}$ be a sequence of real numbers. Consider the following assertions:

(A) If $|X_n + 2 - X_{n+1}| \leq \frac{1}{2}\left|x_{n+1} - X_n\right|$ for all $n \geq 1$, then $\{X_n\}$ is a Cauchy sequence.

(B) If $\{X_n\}$ is bounded, then it Contains a Cauchy Subsequence.

(C) If $\{X_n\}$ is unbounded, then it cannot Contain a Convergent Subsequence.

(D) If $\{X_n\}$ is bounded, and a Subsequence. Of $\{x_n\}$ Converges to a real number L, then $\{x_n\}$ also converges to L which of the above assertions is/are true?

(1) A only (2) A and B only

(3) B and C only (4) D only

39. Consider the following assertions:

(A) sup $\{|x| \sin x : x \in \mathbb{R}\} = \infty$ and inf $\{|x|$ sin x : $x \in \mathbb{R} = 0$

(B) Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 2x +$ 3.Then, Sup $\{f(\sin(x) + 5) : x \in \mathbb{R}\} = 15$ and inf $(f(\sin(x) + 5): x \in \mathbb{R}\} = 11$

(C) Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 5x +$ 5.Then, inf $\{f\left(sin\left(\frac{1}{n}\right)\right): n \geq 1\} = 0$

(D) Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 3x +$ 4.Then, Sup $\{f(f(x): x \in (0,2)\} = 34$

Which of above assertions is /are correct?

(1) B and D only (2) B, C and D only

(3) A, C and D only (4) A and D only

40. $\lim\limits_{x\to 0^+} \frac{x^x - 1}{x}$ is equal to

(1) 0 (2) -1

(3) $-e$ (4) $-\infty$

41. Let $\{C_n\}_{n\in\mathbb{N}}$ be a sequence of none empty connected subsets of $\mathbb{R}^2$ such that $C_{n+1} \subset C_n, n \in \mathbb{N}$. Consider the following statements:

A. $\bigcap_{n=1}^{\infty} C_n$ is non empty

B. $\bigcap_{n=1}^{\infty} C_n$ is connected.

Which of the following is necessarily correct?

(1) A is true but B is false.

(2) B is true but A is false.

(3) Both A and B are true.

(4) Both A and B are false.

42. Let $\{a_n\}_{n\in\mathbb{N}} \subset \mathbb{R}^2$ be such that $a_n = (\cos n, \sin n), n \in \mathbb{N}$. Which of the following statements is correct?

(1) $\{a_n\}_{n\in\mathbb{N}}$ is convergent.

(2) $\{a_n\}_{n\in\mathbb{N}}$ is not convergent but it has a convergent subsequence.

(3) $\{a_n\}_{n\in\mathbb{N}}$ has no convergent subsequence.

(4) $\{a_n\}_{n\in\mathbb{N}}$ is unbounded.

43. The value of $\lim\limits_{x\to\infty}\left[(x^{10} - x^9)^{\frac{1}{10}} - (x^8 - x^7)^{\frac{1}{8}}\right]$ is

(1) 0 (2) 1/8

(3) 1/10 (4) 1/40

44. suppose $f :]-\frac{\pi}{2}, \frac{\pi}{2}[\to \mathbb{R}$ is continuous function such that $f(x) = \frac{\sqrt{9-z}-\sqrt{9+x}}{\sin x \cos x}$ for all $x \neq 0$. Then the value of $f(0)$ is

(1) $-1/9$ (2) $-1/6$

(3) $-1/3$ (4) 0

45. For $a_n = n^{50} e^{-\sqrt{n}}, n \in \mathbb{N}_1$ the sequence $\{a_n\}$ is
(1) Unbounded
(2) Bounded and convergent to a positive real number
(3) Bounded but does not converge
(4) Bounded and convergent to 0

46. Which of the following statements is not re
(1) Every bounded sequence of real number has a monotonic subsequence,
(2) If $\{x_n\}$ is a bounded sequence of real number with $sup_n x_n \neq x_m, \forall m \in \mathbb{N}$, then it has a subsequence converging to $sup_n x_n$
(3) Every unbounded sequence of real number has a monotone subsequence
(4) Every monotonic sequence of real number has a convergent subsequence

47. Let $\{x_n\}$ and $\{y_n\}$ be two sequence defined as

$x_1 = 2, x_{n+1} = \frac{1}{2}\left(x_n = \frac{2}{x_n}\right)$

$y_1 = 2, y_{n+1} = y_n + \frac{1}{n+1}$

For all $n \geq 1$ Then which of the following is true
(1) $\{x_n\}$ and $\{y_n\}$ both are increasing and convergent
(2) $\{x_n\}$ and $\{y_n\}$ both are convergent with $lim x_n < lim y_n$ is monotone but not convergent
(3) $\{x_n\}$ is monotone and convergent where as y_n is monotone but not convergent
(4) $\{x_n\}$ is monotone but not convergent where as y_n is monotone and convergent

48. Evaluate:

$\lim_{n\to\infty}\left(\sqrt{n^2+n} - \sqrt{n^2+1}\right)$

(1) 1
(2) 2
(3) 3
(4) 4

49. Evaluate:

$\lim_{\theta\to\frac{\pi}{2}}(1 - 5\cot\theta)^{\tan\theta}$.

(1) 1/e
(2) 2/e
(3) 3/e
(4) 4/e

50. Evaluate: $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\dots\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$
(1) 1/e
(2) 2/e
(3) 3/e
(4) 4/e

51. Evaluate:

$\lim_{n\to\infty}\frac{1}{n}\left(\prod_{k=1}^{n}(n+k)\right)^{\frac{1}{n}}$.

(1) 1/e
(2) 2/e
(3) 3/e
(4) 4/e

52. Let $a > 0$ be a real number and let $k \in \mathbb{N}$. Evaluate:

$\lim_{n\to\infty}\frac{\left(a+\frac{1}{n}\right)^n\left(a+\frac{2}{n}\right)^n\dots\left(a+\frac{k}{n}\right)^n}{a^{nk}}$

(1) $K!/a^k$
(2) K/a^k
(3) a
(4) a^k

53. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two real sequences. Which of the following statements are true?
(1) If the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent, then the series $\sum_{n=1}^{\infty} a_n b_n$ is convergent.
(2) If the series $\sum_{n=1}^{\infty} a_n$ is convergent and if the series $\sum_{n=1}^{\infty} b_n$ is absolutely convergent, then the series $\sum_{n=1}^{\infty} a_n b_n$ is absolutely convergent.
(3) If $a_n \geq 0$ for all n and is such that the series $\sum_{n=1}^{\infty} a_n$ is convergent, and if the sequence $\{b_n\}_{n=1}^{\infty}$ is bounded, then the series $\sum_{n=1}^{\infty} a_n b_n$ is absolutely convergent.
(4) None of the above

54. Let $\{f_n\}$ be a sequence of functions defined on [0, 1]. Determine $f(x) = lim_{n\to\infty} f_n(x)$, and determine if the following claim is true or false:

$\lim_{n\to\infty}\int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

$f_n(x) = n^2 x(1-x^2)^n$.

(1) True
(2) False
(3) Cannot be decided
(4) None of the above

55. Let $f_n(x) = n^2 x(1-x)^n$. Evaluate:

$\lim_{n\to\infty} f_n(x)$

for $0 < x < 1$.

(1) 0
(2) 1
(3) 2
(4) 3

56. Which of the following functions are continuous at every point of their respective domains of definition?
(1) $f(x) = \lim_{n\to\infty}\frac{x^2 e^{nx}+x}{e^{nx}+1}, x \in \mathbb{R}$.
(2) $f(x) = \lim_{n\to\infty}\frac{x^2 e^{nx}+\cos x}{e^{nx}+1}, x \in \mathbb{R}$.
(3) $f(x) = \lim_{n=>\infty}\frac{1}{n}\log(e^n + x^n), x \geq 0$.
(4) None of the above

57. Which of the following functions are uniformly continuous over their respective domains of definition?

(1) $f(x)=\sum_{n=1}^{\infty}\frac{g(x-n)}{2^n}, x\in\mathbb{R}$,
where $g:\mathbb{R}\to\mathbb{R}$ is a bounded and uniformly continuous function.

(2) $f(x)=\cos x\cos\frac{\pi}{x}, x\in]0,1[$.

(3) Both (1) & (2)

(4) None of the above

58. For the function $f(x)$ on the real line $\mathbb{R}$ defined below, which of the following statements about f is true? Choose all the correct options: $f(x):=\sum_{n\geq 1}\frac{\sin(x/n)}{n}$

(1) f is continuous but not uniformly continuous on $\mathbb{R}$.

(2) f is uniformly continuous on $\mathbb{R}$.

(3) f is differentiable on $\mathbb{R}$.

(4) f is an increasing function on $\mathbb{R}$.

59. For n a positive integer, let $f_n(x)$ be the continuous function $1/(1+nx)$ with domain the positive real numbers. Let $f(x)$ be the pointwise limit of the sequence $\{f_n(x)\}_{n\geq 1}$ of functions. On which of the following intervals is the convergence $f_n\to f$ not uniform? Choose all the correct options:

(1) $(0,1)$ (2) $(1,2)$

(3) $(2,\infty)$ (4) None of the above.

ANSWER KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	2	1	2	2	2	3	3	1	1
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
4	3	4	1	3	3	4	3	3	4
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
4	3	4	4	3	3	1	4	4	3
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
2	3	4	4	4	4	3	2	4	4
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
4	2	3	3	4	1	4	1	4	4
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
4	1	3	2	1	1	1	3	1	

$\frac{\partial L}{\partial a_n} = 2a_n + \lambda = 0 \Rightarrow a_n = \frac{-\lambda}{2}$

$\frac{\partial L}{\partial \lambda} = a_1 + a_2 + \lambda = a_n - 1 = 0$

$\Rightarrow a_1 + a_2 + \cdots + a_n = 1$

$\frac{-\lambda}{2} + \frac{-\lambda}{2} + \cdots + \frac{-\lambda}{2} = 1 \Rightarrow \frac{-n\lambda}{2} = 1$

$\lambda = -\frac{2}{n}$

$\Rightarrow a_1 = \frac{\left(-\frac{2}{n}\right)}{2} = \frac{1}{n}, a_2 = \frac{1}{n}, \ldots a_n = \frac{1}{n}$

$\sum a_k^2 = \frac{1}{n^2} + \frac{1}{n^2} + \cdots + \frac{1}{n^2}$

$= n \times \frac{1}{n^2}$

$= \frac{1}{n}$

$\Rightarrow$ option (2) is correct

5. **Explanation:**

$\{a_n\} = \left(1+\frac{1}{n}\right)^n \qquad \{b_n\} = \left(1+\frac{1}{n}\right)^{n+1} \; n \in \mathbb{N}$

$a_n \to e \qquad b_n \to e$

Let $n = 1 \qquad (1+1)^1 = 2$

$n = 3 \qquad (1+0.5)^2 = (1.5)^2 = 2.25$

$(1+0.33)^3 = (1.33)^3 = 2.35$

$\Rightarrow \{a_n\}$ & $\{b_n\}$ both seq. are monotone increasing

$\Rightarrow$ option (1) is correct

6. **Explanation:**

$a_1 = 1$ for $n \geq 1$

$a_{n+1} = a_n + (-1)^n 2^{-n} \qquad b_n = \frac{2a_{n+1} - a_n}{a_n}$

$a_2 = 1 + (-1)^1 2^{-1}$

$= 1 - {}^1/_2 = {}^1/_2$

$a_3 = 1 - \frac{1}{2} + \frac{1}{2^2}$

$\vdots$

$a_n = \frac{a(1-r^n)}{1-r} \qquad r = -\frac{1}{2}$

$r < 1$

$a_n = \frac{1\left(1-\left(-{}^1/_2\right)^n\right)}{1-\left(-\frac{1}{2}\right)}$

$\lim\limits_{n\to\infty} a_n = \frac{1(1-0)}{{}^3/_2} = {}^2/_3$

$\{a_n\}$ converges to non-zero $= \frac{2}{3}$

$b_n = \frac{2a_{n+1} - a_n}{a_n}$

$\lim\limits_{n\to\infty} b_n = \frac{2l - l}{l} = \frac{l}{l} = 1$

$\Rightarrow \{b_n\}$ is converges

$\Rightarrow \{b_n\}$ is a Cauchy sequences

$\Rightarrow$ (Every convergent seq. is cauchy)

$\Rightarrow$ option (2) is correct

7. **Explanation:**

$\{(-1)^n\}_{n=1}^{\infty} = \{-1, 1, -1, 1, \ldots\}$

Given seq. has upper bound 1 & lower bound – 1

$\therefore \{a_n\}$ is bounded & oscillate

$\Rightarrow$ option (3) is correct

8. **Explanation:**

$\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}, \{c_n\}_{n=1}^{\infty}$ s.t $b_n = a_{2n}$ $c_n = a_{2n+1}$ & $\{a_n\}_{n=1}^{\infty}$ is convergent

$\therefore \{a_{2n}\}$ & $\{a_{2n+1}\}$ are complementary subseq. Of $\{a_n\}$

$\{a_n\}$ converges then $\{a_{2n}\}$ & $\{a_{2n+1}\}$ both are convergent

$\Rightarrow \{b_n\}$ & $\{c_n\}$ both are convergent

$\Rightarrow$ option (3) is correct

9. **Explanation:**

$\lim\limits_{n\to\infty} (2^n + 3^n)^{1/n} = l$

We know that

$\lim\limits_{n\to\infty} (a^n + b^n)^{1/n} = \max\{a, b\}$

$\lim\limits_{n\to\infty} (2^n + 3^n)^{1/n} = \max\{2, 3\}$

$= 3$

$\Rightarrow$ option (2) is correct

10. **Explanation:** Every bounded sequence is not convergent because $\{(-1)^n\}_{n=1}^{\infty}$ is bounded sequence but not convergent

$\Rightarrow$ option (3) is correct

11. **Explanation:**

$a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}$

The sequence then of seq.

If $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} a_n = L$ & there exist an integer N such that

$b_n \leq a_n \leq c_n$ for all $n > N$ then

$\lim\limits_{n\to\infty} a_n = l$

$b_n = \frac{1}{(n+n)^2} + \frac{1}{(n+n)^2} + \cdots + \frac{1}{(n+n)^2} \leq \frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(n+n)}$

$= \frac{1}{2n^2}$ n-times $\leq \frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}$

$b_n = \frac{1}{4n^2} n \leq \frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}$

$\frac{1}{4n} \leq \frac{1}{n^2} + \frac{1}{(n-1)^2} + \cdots + \frac{1}{(2n)^2}$

Now can

$\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2} \leq \frac{1}{n^2} + \frac{1}{n^2} + \cdots + \frac{1}{n^2} = c_n$

n-times

$$\Rightarrow \frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2} \le n.\frac{1}{n^2} = c_n$$

$$\Rightarrow \frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2} < \frac{1}{n} = c_n$$

$$\Rightarrow b_n \le \frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2} \le a_n$$

Where $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} c_n = 0$

$$\lim_{n\to\infty} \frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2} = 0$$

$$\lim_{n\to\infty} \frac{1}{n^2}+\frac{1}{(n+1)^2}+\cdots+\frac{1}{(2n)^2} = 0$$

⇒ given series is converges to zero

⇒ option (1) is correct

12. Explanation:

$a_n = \sin\frac{1}{n^2}$ $\qquad n = 1,2,\ldots$

$\lim\limits_{n\to\infty} \frac{a_n}{b_n}$ $\qquad$ take $b_n = {}^{1}/_{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin {}^{1}/_{n^2}}{{}^{1}/_{n^2}} = 1 \ne 0$$

$\sum b_n = \sum \frac{1}{n^2}$ is convergent

$\therefore \sum \sin\frac{1}{n^2}$ is convergent

⇒ option (2) is correct

13. Explanation:

$$\{a_n\} = \left\{\frac{1}{n}\right\}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

The seq^n $\{a_n\}$ ha supper bound 1 & lower bound o and sec is decreasing

$\therefore \left\{\frac{1}{n}\right\}$ seq is convergent.

⇒ option (1) is correct

14. Explanation:

$$\lim_{n\to\infty} \frac{2n-3}{n+1} = \lim_{n\to\infty} \frac{n\left[2-\frac{3}{n}\right]}{n\left[1+\frac{1}{n}\right]}$$

$$= \lim_{n\to\infty} \frac{2-\frac{3}{n}}{1+\frac{1}{n}}$$

$$= \frac{2-0}{1+0}$$

$l = 2$

⇒ option (3) is correct

15. Explanation:

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right) \to 1^\infty \text{ form}$$

$$l = e \lim_{n\to\infty} n\left[1+\frac{1}{n}-1\right]$$

$l = e$

⇒ option (1) is correct

16. Explanation:

$\{(-1)^n\} = \{a_n\}$

Given seq is bounded but not cgt.

∴ Every bounded seq is convergent statement is false.

⇒ option (1) is correct

17. Explanation:

Limit points of $\{(-1)^n\}$ are {1,-1}

So upper limit of $\{(-1)^n\}$ is 1

18. Explanation:

$$\langle a_n\rangle = \frac{e^n}{n}$$

$$a_1 = \frac{e}{1}, a_2 = \frac{e^2}{2}, a_3 = \frac{e^3}{3}, \ldots$$

We can see that

$a_1 < a_2 < a_3 < \ldots$

∴ $\langle a_n\rangle$ is monotonically increasing sequence

And $\lim\limits_{n\to\infty} \frac{e^n}{n} = \lim\limits_{n\to\infty} \frac{e^n}{1} = \infty$

∴ The sequence is not bounded above

⇒ not bounded sequence

19. Explanation:

Solution: $\langle x_n\rangle = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$

As SOPS is greater than 1

∴ The given series is divergent and we know that

A sequence is cg tiff Cauchy but the sequence is divergent

⇒ does not satisfy Cauchy criteria

20. Explanation:

Solution: $\lim\limits_{n\to\infty} 2^{2n}\left(1-\cos\frac{1}{2^n}\right)$

$$= \lim_{n\to\infty} \frac{2^{2n}}{\frac{1}{\left(1-\cos\frac{1}{2^n}\right)}}$$

$$= \lim_{n\to\infty} \frac{(2^{2n-1})(2n)}{\frac{-1}{\left(1-\cos\frac{1}{2^n}\right)^2}\sin\left(\frac{1}{2^n}\right)\cdot n2^{n-1}}$$

$= DNA$

⇒ does not converge

21. Explanation:

Let $\{a_n\}$ be bounded sequence of real number.

There is a sub sequence of $\langle a_n\rangle$ which is convergent

22. Explanation:

Let $a_n = (-1)^n$

Is bounded but not convergent

[PRACTICE QUESTIONS]

1. **Explanation:**

$\{a_n\} = \left\{\cos\frac{n\pi}{4} + \sin\frac{n\pi}{4}\right\} n \in \mathbb{N}.$

n –odd, n –even

$n = 2m$ for some $m \in \mathbb{N}$

$a_n = a_n = a_{2m} = \cos\left(2m\frac{\pi}{4}\right) + \sin\left(2m\frac{\pi}{4}\right)$

$= \cos\left(m\frac{\pi}{2}\right) + \sin\left(m\frac{\pi}{2}\right).$

m-even $\cos\left(even\ \frac{\pi}{2}\right) + \sin\left(even\frac{\pi}{2}\right)$

$m = 2k \qquad \therefore cso\ \left(2k\frac{\pi}{2}\right) + \sin\left(2k\frac{\pi}{2}\right) = \cos(k\pi) +$ $\sin(k\pi)\,.\,\forall k \in \mathbb{N}.$

$\cos(k\pi) \notin (+1,-1)\ \sin(k\pi) = 0\ \forall k \in \mathbb{N}$

m – even $\cos\left(m\frac{\pi}{2}\right) + \sin\left(m\frac{\pi}{2}\right), \in \{1,-1\}$

m –odd $m = 2k+1$

$a_n = a_{2(2k+1)} = \cos\left((2k+1)\frac{\pi}{2}\right) + \sin\left((2k+1)\frac{\pi}{2}\right)$

$\cos\left((2k+1)\frac{\pi}{2}\right) = 0\ \sin\left((1k+1)\frac{\pi}{2}\right) \in (1,-1)$

n –odd $n = 2m+1, m \in \mathbb{N}.$

$a_n = a_{2m+1} = \cos\left((2m+1)\frac{\pi}{4}\right) + \sin\left((2m+1)\frac{\pi}{4}\right)$

$\in \{\sqrt{2}, -\sqrt{2}\}$

$\cos\left((2m+1)\frac{\pi}{4}\right) \in \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$

$\sin\left((2m+1)\frac{\pi}{4}\right) \in \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$

$a_n \in [1,-1,0,\sqrt{2},-\sqrt{2}]$

$\overline{lim} = \sqrt{2} \quad \underline{lim} = -\sqrt{2}$

Option (1) is correct option

2. **Explanation:**

$\{a_n\}_{n=1}^{\infty}\ a_n \in (-1,1) = A$

Recall : (x_n) in x then the set of limit pt of (x_n) are in $\overline{x}$

$\overline{A} = \overline{(-1,1)} = [-1,1] = \overline{A}$

Every limit pt of (a_n) in $[-1,1]$

⇒ option (2) is correct

3. **Explanation:**

$\lim\limits_{x\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right]$

$n^2+1 \le n^2+2, n^2+2, \ldots .n^2+n \le n^2+n$

$\sqrt{n^2+1} \le \sqrt{n^2+1}, \sqrt{n^2+2}, \ldots .\sqrt{n^2+n} \le \sqrt{n^2+n}$

$\frac{1}{\sqrt{n^2+1}} \ge \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}} \ge \frac{1}{\sqrt{n^2+n}}$

$\frac{n}{\sqrt{n^2+1}} \ge \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{n}{\sqrt{n^2+n}} \ge \frac{n}{\sqrt{n^2+n}}$

$\frac{n}{\sqrt{n^2+1}} \ge a_n \ge \frac{n}{\sqrt{n^2+n}}$

$\lim\limits_{n\to\infty}\frac{n}{\sqrt{n^2+1}} = \lim\limits_{n\to\infty}\frac{n}{\sqrt{n^2+n}} = 1$

$\lim\limits_{n\to\infty} a_n = 1$

⇒ option (1) is correct

4. **Explanation:**

$a_n\left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = (b_n)^{\frac{1}{2}}$

$\lim\limits_{n\to\infty}\frac{b_{n+1}}{b_n} = \lim\limits_{n\to\infty} a_n$

$\lim\limits_{n\to\infty}\frac{b_{n+1}}{b_n} = \lim\limits_{n\to\infty}\frac{(n+1)^{n+1}}{(n+1)!} \times \frac{n!}{n^n}$

$= \lim\limits_{n\to\infty}\frac{(n+1)^n(n+1)\times n!}{(n+1)n!\times n^n}$

$= \lim\limits_{n\to\infty}\frac{(n+1)^n}{n^n}$

$\lim\limits_{n\to\infty}\left(\frac{n+1}{n}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$= e^{\lim\limits_{n\to\infty}\left[1+\frac{1}{n}-1\right]}$

$\lim\limits_{n\to\infty} a_n = e$

⇒ option (2) is correct

5. **Explanation:**

For $a_1 > 0$ seq $\{a_n\}_{n=1}^{\infty}$

$a_{n+1} = 1 + \frac{1}{a_n}$ is converges

$\lim\limits_{n\to\infty} a_n - l, ''y \lim\limits_{n\to\infty} a_{n+1} = l$

$l = 1 + \frac{1}{l}$

$l = \frac{l+1}{l}$

$l^2 = l + 1$

$l^2 - l - 1 = 0$

$l = \frac{1\pm\sqrt{1+4}}{2} = \frac{1\pm\sqrt{5}}{2} = \frac{1+\sqrt{5}}{2}$

⇒ option (2) is correct

6. **Explanation:**

$$f(x) = \begin{cases} \frac{1}{a^{n-1}} & \frac{1}{a^n} < x \le \frac{1}{a^{n-1}} \\ 0 & x = 0 \end{cases}$$

$f(x) = \frac{1}{a^{n-1}} \frac{1}{a^n} < x < \frac{1}{a^{n-1}}$

$f(x) = \frac{1}{a^0}$ when $\frac{1}{a'} < x < \frac{1}{a^0}$

$f(x) = \frac{1}{a}$ when $\frac{1}{a^2} < x < \frac{1}{a^1}$

$f(x) = \frac{1}{a^2}$ when $\frac{1}{a^3} < x < \frac{1}{a^2}$

Clearly this function is discontinuous at the $pt\ 0, \frac{1}{a}, \frac{1}{a^2}, \ldots$

The set of pt of discontinuity has finite limiting pt.

$f \in \mathbb{R}[0,1]$

Now,

$$\int_{\frac{1}{a^{n+1}}}^{1} f(x)dx = \int_{\frac{1}{a^{n+1}}}^{1/2^n} f(x)dx + \cdots + \int_{\frac{1}{a^3}}^{\frac{1}{a^2}} f(x)dx +$$

$$\int_{\frac{1}{a^2}}^{\frac{1}{a}} f(x)dx + \int_{\frac{1}{a}}^{1}$$

$$= \int_{\frac{1}{a^{n+1}}}^{\frac{1}{a}} \frac{1}{a^n}dx + \cdots + \int_{\frac{1}{a^3}}^{\frac{1}{a}} \frac{1}{a^2}dx + \int_{\frac{1}{a^2}}^{\frac{1}{a}} \frac{1}{a}dx$$

$$+ \int_{\frac{1}{a}}^{1} 1dx$$

$$= \frac{1}{a^n}[x]_{\frac{1}{a^{n+1}}}^{\frac{1}{a^n}} + \cdots + \frac{1}{a^2}[x]_{\frac{1}{a^3}}^{\frac{1}{a^2}} + \frac{1}{a}[x]_{\frac{1}{a^2}}^{\frac{1}{a}} + [x]_{\frac{1}{a}}^{1}$$

$$= \frac{1}{a^n}\left[\frac{1}{a^n} - \frac{1}{a^{n+1}}\right] + \cdots + \frac{1}{a^2}\left[\frac{1}{a^2} - \frac{1}{a^3}\right]$$

$$+ \frac{1}{a}\left[\frac{1}{a} - \frac{1}{a^2}\right] + \left[1 - \frac{1}{a}\right]$$

$$= \left[1 + \frac{1}{a^2} + \frac{1}{a^4} + \cdots + \frac{1}{a^{2n}}\right]$$

$$= \left[\frac{1}{a} + \frac{1}{a^3} + \cdots + \frac{1}{a^{2n+1}}\right]$$

$$= 1\left[1 - \left(\frac{1}{a^2}\right)^n\right] = \frac{1}{a}\frac{\left[1-\left(\frac{1}{a^2}\right)^n\right]}{1-\frac{1}{a^2}}$$

$$= 1 - \frac{\frac{1}{a^{2n}}}{1-\frac{1}{a^2}}$$

$$= \frac{1-\frac{1}{a^{2n}}}{\frac{a^2-1}{a^2}} - \frac{\frac{1}{2}\left[1-\frac{1}{2^{2n}}\right]}{\frac{a^{2n}-1}{a^{2n}}}$$

$$= \frac{a^2}{a^2+1}\left(1 - \frac{1}{a^{2n}}\right)$$

$$\lim_{n\to\infty} \int_{\frac{1}{a^n}}^{1} f(x)dx = \frac{a}{a+1}\left(1 - \frac{1}{2^{2n}}\right)$$

$$= \frac{a}{a+1}(1-0).$$

$$\lim_{n\to\infty} \int_0^1 f(x)dx = \frac{a}{a+1}$$

⇒ option (2) is correct

7. Explanation:

$\{a_n\}_{n=1}^{\infty}$ be seq. of real no.

$\lim\limits_{n\to\infty} a_n = l$ and $\lim\limits_{n\to\infty} b_n = m \quad l, m \in \mathbb{R}$

$l = \lim\limits_{n\to\infty} \frac{1}{n}(a_1b_n + a_2b_{n-1} + \cdots + a_nb_1)$

Which is cesaro's theorem

So limit

$$= \lim_{n\to\infty} \frac{1}{n}(a_1b_n + a_2b_{n-1} + \cdots + a_nb_1)$$

$l.m$

⇒ option (3) is correct

8. Explanation:

$\lim\limits_{n\to\infty} |x_n| = |l|$ (where $l \in \mathbb{R}$)

Then sequence $\{x_n\}_{n=1}^{\infty}$ may not be converge to l

⇒ for example

$\lim\limits_{n\to\infty} |(-1)^n| = 1 \quad 1 \in \mathbb{R}$

But sequence $\{(-1)^n\}$ not convergent

⇒ option (3) is correct

9. Explanation:

If x_0 is a limit point of a sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers, then there exists a subsequence of $\{x_{nk}\}_{k=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$, which converges to x_0

10. Explanation:

$$x_{n+1} = 1 + \sqrt{x_n}$$

$$x_{n+1} - x_n = 1 + \sqrt{x_n} - 1 - \sqrt{x_n - 1}$$

$$\sqrt{x_n} - \sqrt{x_n - 1} > 0$$

$$x_{n+1} - x_n > 0$$

∴ sequence $\{x_n\}_{n=1}^{\infty}$ is monotonically increasing.

We check limit

$$\lim_{x\to\infty} x_n = l$$

$$x_{n+1} = 1 + \sqrt{x_n}$$

$$l = 1 + \sqrt{l}$$

$$l - 1 = \sqrt{l}$$

$$l^2 - 2l + 1 = l \Rightarrow l^2 - 3l + 1 = 0$$

$$l = \frac{3\pm\sqrt{9-4}}{2}$$

$$l = \frac{3\pm\sqrt{5}}{2} l = \frac{3+\sqrt{5}}{2}$$

∴ $\{x_n\}_{n=1}^{\infty}$ is convergent $\frac{3+\sqrt{5}}{2}$.

∴ ⇒ option (1) is correct.

11. Explanation:

$$x_1 = \frac{1}{3}$$

$$x_{2n} = \frac{1}{2}, x_{2n-1}\ x_{2n+1} = \frac{1}{3} + x_{2n}$$

$$n = 1\ x_2 = \frac{1}{2}x_1\ x_3 = \frac{1}{3} + x_2$$

$$x_2 = \frac{1}{6}\ x_3 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$n = 2$$

$$x_4 = \frac{1}{2}x_3\ x_5 = \frac{1}{3} + x_4$$

$$= \frac{1}{2}, \frac{1}{2} = \frac{1}{4} = \frac{1}{3} + \frac{1}{4}$$

$$= \frac{7}{12}$$

$$n = 3$$

$$x_6 = \frac{1}{2}x_5\ x_7 = \frac{1}{3} + x_6$$

$$= \frac{7}{24} = \frac{1}{3} + \frac{7}{24}$$

$$= \frac{45}{72}$$

$$x_n = \left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \frac{1}{4}, \frac{7}{12}, \frac{7}{24}, \frac{45}{72} \cdots\right\}$$

$$\lim\sup\{x_n\} = \frac{1}{2}\ \lim\inf\{x_n\} = \frac{1}{6}$$

⇒ option (4) is correct

12. Explanation:

1. $\{a_n\} = \left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$

   $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is converges to e

2. $\{a_n\} = \left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$

   $\lim\limits_{n\to\infty}\frac{(-1)^n}{n}$ is converges to 0

3. $\left\{1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right\}$ seq is converges

4. $\{a_n\} = \left\{\frac{2^n}{n^n}\right\} n^n > 2^n \therefore \lim\limits_{n\to\infty} a_n = 0$

   $\therefore \{a_n\}$ is convergent

13. Explanation:

$\therefore \{\alpha_n\}$ is seq of real no.

α be a non-zero real no.

$S.t \lim\limits_{n\to\infty}(a_1+a_2+\cdots a_n) = \alpha$

Then $\lim\limits_{n\to\infty}(\alpha_n) = \alpha$?

$\beta_n = \alpha_1+\alpha_2+\cdots+\alpha_n$

$\beta_{n-1} = a_1+a_2+\cdots+a_{n-1}$

$\beta_n-\beta_{n-1} = a_1+a_2+\cdots.+a_n+a_1-a_2\ldots a_{n-1}$

$\beta_n-\beta_{n-1} = a_n$

$\lim\limits_{n\to\infty}\beta_n - \lim\limits_{n\to\infty}\beta_{n-1} = \lim\limits_{n\to\infty} a_n$

$\alpha-\alpha = \lim\limits_{n\to\infty} a_n$

$\therefore \lim\limits_{n\to\infty} a_n = 0$

⇒ option (4) is correct.

14. Explanation:

$a_n = 2^n+n^2$ for $n \le 100$ and $a_n = 3+(-1)^n\frac{n^\wedge}{2^n}$ for n >100

is convergent sequence and R is complete

So, every convergent sequence is Cauchy.

15. Explanation:

Option (1) is true statement, as if $\langle x_n\rangle$ be any sequence, then by monotone subsequence theorem, $\langle x_n\rangle$, have a monotone subsequence say $\langle x_{n_k}\rangle$.

Since, $\langle x_n\rangle$ is bounded, so is subsequence, hence $\langle x_{n_k}\rangle$ is convergent (being monotone and bounded)

Option (2) is true statement, as it is statement of monotone subsequence theorem

Option (4) is true statement, as by definition A sequence is a function having domain set ℕ of natural numbers So, Range set have countable number of terms.

Option (3) is wrong, as if we assumed sequence $\langle x_n\rangle$ as $x_n = n$, then clearly $\langle x_n\rangle$ have no limit point.

Hence option (A), (B) and (D) are correct

16. Explanation:

Since, $\langle a_n\rangle = \langle 1,1,\frac{1}{2},1,\frac{1}{2},\frac{1}{3},\ldots\rangle$

Clearly, each of the number of the form $\frac{1}{n} : n\in\mathbb{N}$, repeated infinitely many times in this sequence hence each of the number of the form $\left\{\frac{1}{n} : n\in\mathbb{N}\right\}$ is a limit point of $\langle a_n\rangle$

Therefore both (1) and (4) are correct statement.

Also, $\lim\limits_{n\to\infty}\sup a_n = \sup\left\{\frac{1}{n} : n\in\mathbb{N}\right\} = 1$

And, $\lim\limits_{n\to\infty}\inf a_n = \inf\left\{\frac{1}{n} : n\in\mathbb{N}\right\} = 0$

Therefore, options (2) and (3) are correct.

Hence options (1), (2), (3) and (4) are correct.

17. Explanation:

Clearly, $F_n = \left[\frac{-1}{n},\frac{1}{n}\right]$, we define, $F = \cap_{n=1}^{\infty} F_n$

claim: $F = \{0\}$

Since, for every $n\in\mathbb{N}$, 0 is an element of the set $\left[\frac{-1}{n},\frac{1}{n}\right]$

$\therefore 0 \in \left\{x\in\mathbb{R} \mid x\in\left[\frac{-1}{n},\frac{1}{n}\right] \text{for all } n\in\mathbb{N}\right\} = \cap_{n=1}^{\infty}\left[\frac{-1}{n},\frac{1}{n}\right]$,

Clearly, $\{0\} \subseteq \cap_{n=1}^{\infty}\left[\frac{-1}{n},\frac{1}{n}\right] = F \Rightarrow \{0\}\subseteq F$

Now, to show that $F\subseteq\{0\}$, let $x\in F$ is arbitrary, then $x\in\left[\frac{-1}{n},\frac{1}{n}\right]$ for every $n\in\mathbb{N}$

Suppose that $x\neq 0$, then $|x|>0$ and by archemedian property of real numbers there exist natural number N such that00 $< \frac{1}{N} < |x|$, therefore $x\notin\left[\frac{-1}{N},\frac{1}{N}\right]$

This gives us a contradiction, hence $x = 0$. Since, x is arbitrary we have, $F \subseteq \{0\}$, therefore, $F = \{0\}|$, since, singleton sets are closed

Hence option (1), (2), (3) and (4) are correct.

18. Explanation:

Here, $X_n = \frac{2^{n+1}3^{n+1}}{2^n+3^n}\ \forall\, n\in\mathbb{N}$

$$= \frac{\left(1+\left(\frac{2}{3}\right)^{n+1}\right)3^{n+1}}{\left(1+\left(\frac{2}{3}\right)^{n}\right)3^{n}} = 3.\frac{1+\left(\frac{2}{3}\right)^{n+1}}{1+\left(\frac{2}{3}\right)^{n}}$$

$$\therefore \lim_{n\to\infty} X_n = 3.\frac{1+0}{1+0} = 3\left(\text{as} \lim_{n\to\infty}\left(\frac{2}{3}\right)^n = 0\right)$$

Hence option (3) is correct.

19. Explanation:

Consider,

$$x_n = \sqrt{(n+1)(n+2)} - n$$

$$= \frac{(\sqrt{(n+1)(n+2)}-n)(\sqrt{(n+1)(n+2)}+n)}{(\sqrt{(n+1)(n+2)}+n)} = \frac{(n+1)(n+2)-n^2}{\sqrt{(n+1)(n+2)}+n}$$

$$= \frac{3n+2}{n\left(\sqrt{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}+1\right)} = \frac{3n+\frac{2}{n}}{\left(\sqrt{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}+1\right)}$$

$$\therefore \lim_{n\to\infty} x_n = \frac{3+0}{\sqrt{(1+0)(1+0)}+1} = \frac{3}{2}$$

Hence option (3) is correct.

20. Explanation:

Given that, $x_n = 1 + (-1)^n + \frac{1}{2^n}$

Then, $\langle x_{2n}\rangle = 1 + (-1)^{2n} + \frac{1}{2^n}$

$=1 + 1 + \frac{1}{2^{2n}}$, converges to 2 as $n \to \infty$

Also, $\langle x_{2n}\rangle = (1 + (-1)^{2n+1}) + \frac{1}{2^{2n+1}}$

$= \frac{1}{2^{2n+1}}$ converges to 0 as $n \to \infty$

$\therefore \lim \sup\langle x_n\rangle = 2$ and $\lim \inf\langle x_n\rangle = 0$

Hence option (4) is correct

21. Explanation:

Given that

$x_1 = 2$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$,

Note that for any two real Numbers a, b we have $\sqrt{ab} < \frac{a+b}{2}$

$\therefore \frac{1}{2}\left(x_n + \frac{2}{x_n}\right) - x_n = \frac{1}{x_n} - \frac{x_n}{2}$

$= \frac{2-x_n^2}{2x_n} \leq 0$ as $x_n \geq \sqrt{2} \forall n > 1$

$\Rightarrow x_{n+1} \leq x_n \forall\, n \in \mathbb{N}$

This implies that $\langle x_n\rangle$ is increasing sequence. Also $\langle x_n\rangle$ is bounded below, Hence convergent

Let $\lim x_n = l$ then $\lim x_{n+1} = l$

Therefore

$l = \frac{1}{2}\left(l + \frac{2}{l}\right)$ i.e. $2l = \left(\frac{l^2+2}{l}\right)$

$\Rightarrow l = \sqrt{2}$

Therefore $\langle x_n\rangle$ is decreasing sequence converges to $\sqrt{2}$

Hence Option (4) is correct.

22. Explanation:

Gives that, $\langle x_n\rangle$ be an unbounded sequence in $\mathbb{R}$

i.e. There does not exist any constant such that $M \in \mathbb{R}$ such that $|x_n| < M \forall\, n \in \mathbb{N}$

If $M = 1$, then there exist some element, x_{n_1} such that

$x_{n_1} > 1$

if $M = 2$, then there exist some element, x_{n_2} such that

$x_{n_1} > 2$

Similarly, $x_{n_k} > k$ for $k \in \mathbb{N}$

$\therefore \frac{1}{x_{n_k}} < \frac{1}{k}$

Since, $\lim_{k\to k} \frac{1}{k} = 0$, we have $\lim_{x\to 0} \langle\frac{1}{x_{n_k}}\rangle = 0$

Hence option (3) is correct.

23. Explanation:

Given that $\langle a_n\rangle$ be a bounded sequence, for example, let us consider $\langle a_n\rangle = \langle\frac{1}{n}\rangle$

Then, $\sup\langle a_n\rangle = \sup\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\} = 1$ and

$\inf\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\} = 0$ as $\langle a_n\rangle$ is convergent sequence so,

$\lim_{n\to\infty} \inf a_n = \lim_{n\to\infty} \sup a_n = 0$

$\therefore 1 = \sup\langle a_n\rangle \not< \lim_{n\to\infty} \sup a_n$

So, (A) and (C) is wrong

Also, if we take $\langle a_n\rangle = \langle-\frac{1}{n}\rangle$, is bounded sequence, then $\inf\langle a_n\rangle = -1$ and $\lim_{n\to\infty} \inf a_n = 0$ but $\lim_{n\to\infty} \inf a_n \not< \inf\langle a_n\rangle$, hence (B) is wrong.

Hence option (4) is correct

24. Explanation: neither monotonically increasing nor monotonically decreasing

25. Explanation:

$$\lim_{n\to\infty} x_n = l$$

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \frac{1}{4-x_n}$$

$$l = \frac{1}{4-l}$$

After solving it we get

$l = 2 - \sqrt{3}$

26. Explanation:

Consider the sequence $x_n = 1 + (-1)^n + \frac{1}{3^n}$

Consider the subsequence $x_{2n} = 1 + (-1)^{2n} + \frac{1}{3^{2n}} = 2 + \frac{1}{3^{2n+1}}$

Clearly, as $n \to \infty$, x_{2n} converges to 2.

Also, $x^{2n+1} = 1 + (-1)^{2n+1} + \frac{1}{3^{2n}} = 2 + \frac{1}{3^{2n+1}}$

Clearly, as $n \to \infty$, x_{2n} as, converges to 0.

Hence both 0 and 2 are limit points of this sequence, then $\lim\inf x = 0$ and $\lim\sup \; x_n = 2$

Hence option (4) is correct.

27. **Explanation:**

$\langle x_n \rangle$ is a sequecne of natural number such that $x_n \geq 0 \forall n \in \mathbb{N}$.

Also, $y_n = (-1)^n x_n$, converges.

Since, $\langle y_n \rangle$ converges, so its subsequence $\langle y_{2n} \rangle = \langle x_{2n} \rangle and \langle y_{2n+1} \rangle = -\langle x_{2n+1} \rangle$ also converges and converges to same limit.

Since, $y_{2n} = x_{2n} \geq 0$ for all n, so $\lim y_{2n} \geq$ and $y_{2n+1} = -x_{2n+1} \leq 0$ for all n, so $\lim y_{2n+1} \leq 0$

But both are same, so $\lim y_{2n} = \lim y_{2n+1} = 0$

i.e. $\lim\limits_{n\to\infty} x_{2n} = \lim\limits_{n\to\infty} x_{2n+1} = 0$ So $\langle x_n \rangle$ is also convergent.

Hence option (1) is correct.

28. **Explanation:**

$x_n = \left[\frac{5+n}{2n}\right] \forall n \in \mathbb{N}$

Note that, $5 + n < 2n \; \forall n \geq 6$

$\Rightarrow 0 < \frac{5+n}{2n} < 1 \; \forall \geq 6$

$\therefore x_n = \left[\frac{5+n}{2n}\right] = 0 \; \forall n \geq 6$

Hence $\lim\limits_{n\to\infty} x_n = 0$

Hence option (4) is correct

29. **Explanation:** Since, $f_n(x) = x^n, x \in [0,1] \cup \{2\}$

$$\text{Clearly, } f_n(x) = \begin{cases} 0 \; if \; x = 0 \\ x^n \; if \; 0 < x < 1 \\ 1 \; if \; x = 1 \\ 2'' \; if \; x = 2 \end{cases}$$

Clearly, $f_n(x)$ is not bounded as for large value of n $\langle 2^n \rangle$ is unbounded.

Hence option (4) is correct

30. **Explanation:** $x_n = (-1)^{p_n+1}\left(1 + \frac{1}{p_n}\right)$

For prime numbers greater than 2 we have

$x_n = \left(1 + \frac{1}{p_n}\right)$

$\therefore \lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty}\left(1 + \frac{1}{p_n}\right) = 1$

$\Rightarrow$ option (3) is correct

31. **Explanation:**

$m \notin S, M \notin S.$

32. **Explanation:**

f is taking all values between $-M$ and M

Since, $|f(x)| \leq M$

$-M \leq f(x) \leq M$

33. **Explanation:**

By definition we have

Let $\langle a_n \rangle$ be sequence of real no and $l \in \mathbb{R}$ then for given $\varepsilon > 0, \exists \; n_0 \in \mathbb{N}$ such that $|a_n - l| < \varepsilon$ when ever $n \geq n_0$

$\therefore$ Does not converge to l

$\Rightarrow$ Option (4) is correct

34. **Explanation:**

Given: $\langle a_n \rangle$ is a sequence of positive number and we have

$a_n a_{n+2} \leq a_{n+1}^2$

$\therefore a_1 a_3 \leq a_2^2$

$a_2 a_4 \leq a_3^2$

$a_3 a_5 \leq a_4^2$

$a_4 a_6 \leq a_5^2$

$\vdots$

$a_{n-3} a_{n-1} \leq a_{n-2}^2$

$a_{n-2} a_n \leq a_{n-1}^2$

$a_{n-1} a_{n+1} \leq a_n^2$

$a_n a_{n+2} \leq a_{n+2}^2$

Now multiplying all these terms, we get

$a_1 a_{n+2} \leq a_2 a_{n+1}$

Now let us assume

$a_1 = 2a_2$

$\Rightarrow 2a_2 a_{n+2} \leq a_2 a_{n+1}$

$\Rightarrow 2a_{n+2} \leq a_{n+1} \Rightarrow$ The sequence is monotonically increasing

Let l is the limit

$\Rightarrow 2l \leq l$

$\Rightarrow 2l - l \leq 0$

$\Rightarrow l \leq 0$

But the sequence is of positive number

$\therefore$The sequence is not convergent

$\Rightarrow$ option (4) is correct

35. **Explanation:** Now, as $a_n > 0, \forall n \in \mathbb{N}$

And a_n is monotonically decreasing

$\Rightarrow \langle a_n \rangle$ is convergen

$\Rightarrow$ option (4) is correct

36. Explanation: S_1: It is true by Bolzano Weierstrass theorem

S_2: let $a_n = \begin{cases} 1, & for\ n = odd \\ -n, & for\ n = even \end{cases}$

Let subsequence be

$a_{2n+1} = \{1,1,1,\dots,1\}$

Which is convergent but the sequence is not bdd

⇒ option (4) is correct

37. Explanation:

$$a_n = (-1)^{n-1}\left(1 + \frac{1}{2n+1}\right)$$

$$a_n = \begin{cases} 1 + \frac{1}{2n+1}, & n - odd \\ (-1)\left(1 + \frac{1}{2n+1}\right), & n - even \end{cases}$$

$$\lim_{n\to\infty} a_n = \begin{cases} 1, & n - odd \\ -1, & n - even \end{cases}$$

∴ lim sup $a_n = 1$ and lim inf $a_n = -1$

C) Let $\frac{-1}{n^2} \le a_n \le \frac{n+1}{n^3}$, $\forall n \ge 1$

And let $\frac{-(n+1)}{n^3} \le \frac{-n}{n^3} = \frac{-1}{n^2} \le a_n \le \frac{n+1}{n^3}$

$\Rightarrow |a_n| \le \frac{n+1}{n^3}, \forall n \ge 1$

38. Explanation:

1) The sequence of the form

$|x_{n+2} - x_{n+1}| \le c|x_{n+1} - x_n|$

Where $0 < c < 1$

Is called contractive sequence and c is contractive constant. And every contractive sequence is Cauchy

2) By Bolzano Weieratrass theory

If $\{x_n\}$ is bounded then it has a convergent subsequence and every cgt subsequence is Cauchy. Subsequence

3) Let $a_n = \begin{cases} 2, & n \text{ is odd} \\ n, & \text{otherwise} \end{cases}$

$\langle a_n \rangle$ is unbounded but $\lim_{n\to\infty} a_n = 2$

4) Let $a_n = (-1)^n$ and $a_{2n} = 1$

but $\langle a_n \rangle$ is not convergent

⇒ statement A and B are correct

⇒ option (2) is correct

39. Explanation:

2) Given : $F(x) = 2x + 3$

$F(\sin x + 5) = 2(\sin x + 5) + 3$

$= 2\sin x + 13$

$\therefore \sup\{f(\sin x + 5)\} = 2\sin x + 13$

$= 2 + 13 = 15$

And $\inf\{F(\sin x + 5)\} = 2\sin x + 13$

$= 2(-1) + 13$

$= 11$

⇒ option (2) is correct

3) Given : $F(x) = 5x + 5$

$\sin\frac{1}{n} \quad n \ge 1$

$\sup\left\{F\left(\sin\frac{1}{n}\right)\right\} = 5\sin\frac{1}{n} + 5$

$= 5(\sin 1) + 5$

$5\sin 1 + 5$

$\inf\left\{F\left(\sin\frac{1}{n}\right)\right\} = 5\sin\frac{1}{n} + 5$

$= 0 + 5$

$= 5$

⇒ option (C) is incorrect

4) $F(x) = 3x + 4$

$F(F(x)) = 3(3x + 4) + 4$

$= 9x + 12 + 4$

$= 9x + 16$

$\sup\{F(F(x)) : x \in (0,2)\} = 9 \times 2 + 16$

$= 34$

⇒ option (4) is correct

40. Explanation:

$$\lim_{x\to 0^+} \frac{x^x - 1}{x}$$

$$\lim_{x\to 0^+} x^x \log x = -\infty$$

41. Explanation:

$\{C_n\}$ be sequence of non empty connected subset of $\mathbb{R}^2$ such that

$C_{n+1} C_n, n \in \mathbb{N}$

Let

$C_n = \left(\frac{1}{n}, \frac{-1}{n}\right)$

$\bigcap_{n=1}^{\infty} C_n = \phi$

⇒ which is empty & connected

∴ Both statement is false.

42. Explanation:

Here

$a_n = (\cos n, \sin n)\ n \in \mathbb{N}$

$\lim_{n\to\infty} a_n = ((-1)^n, (-1)^n)$

⇒ $\langle a_n \rangle$ is not convergent

But $\langle a_n \rangle$ has convergent

43. Explanation:

$$\lim_{x\to\infty}\left[(x^{10}-x^9)^{\frac{1}{10}}-(x^8-x^7)^{\frac{1}{8}}\right]$$

$$\lim_{x\to\infty}\frac{\left[[x^{10}-x^y]^{\frac{1}{10}}-(x^8-x^7)^{\frac{1}{8}}\right]\left[(x^{10}-x^9)^{\frac{1}{10}}+(x^8-x^7)^{\frac{1}{8}}\right]}{(x^{10}-x^9)^{1/10}+(x^8-x^7)^{\frac{1}{8}}}$$

$$\lim_{x\to\infty}\frac{(x^{10}+x^9)^{\frac{1}{5}}+(x^8-x^7)^{\frac{1}{4}}}{(x^{10}-x^9)^{\frac{1}{10}}+(x^8-x^7)^{\frac{1}{8}}}=\frac{1}{10}$$

44. Explanation:

$f:\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\to\mathbb{R}$ such that

$$f(x)=\frac{\sqrt{9-x}-\sqrt{9+x}}{\sin x\cos x}$$

$$\lim_{x\to0}f(x)=\lim_{x\to0}\frac{\sqrt{9-x}-\sqrt{9+x}}{\sin x\cos x}$$

$$=\lim_{x\to0}\frac{\frac{-1}{2\sqrt{9-x}}\ \frac{1}{2\sqrt{9+x}}}{\cos 2x}$$

$$=\lim_{x\to0}\frac{-1}{\sqrt{9-x}\cos 2x}$$

$$=\frac{-1}{3}$$

45. Explanation:

$$a_n=n^{50}e^{\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{n^{50}}{e^{-\sqrt{5}}}=0$$

$\langle a_n\rangle$ is convergent

$\Rightarrow\langle a_n\rangle$ is bounded

46. Explanation: Every bounded sequence of real number has a monotonic subsequence.

47. Explanation:

We have

$x_1=2\qquad x_{n+1}=\frac{1}{2}\left(x_n+\frac{2}{x_n}\right)$

$y_1=2\ y_{n+1}=y_n+\frac{1}{n+1}$

$Am\geq 6m$

$$\frac{2_n+\frac{2}{x_0}}{2}\geq\sqrt{x_n\frac{2}{x_n}}$$

$x_{n+1}\geq\sqrt{2}$

$\langle x_n\rangle$ is bounded below

$$x_{n+1}=\frac{x_n}{2}+\frac{1}{x_n}$$

$$x_{n+1}-x_n=\frac{x_n}{2}+\frac{1}{x_n}-x_n=\frac{1}{x_n}-\frac{x_n}{2}$$

$$=\frac{2-xn^2}{2x_n}<0$$

$x_{n+1}-x_n<0$

$\Rightarrow x_{n+1}<x_n$

$\Rightarrow\langle x_n\rangle$ is bounded below & decreasing

$\Rightarrow\langle x_n\rangle$ converges to infinite

(2)

$y_1=2$

$y_{n+1}=y_1+\frac{1}{n+1}$

$y_2=2+\frac{1}{3}\ y_3=2+\frac{1}{3}+\frac{1}{4}$

$\dots y_n=2+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\dots\dots$

$y_1=2$

$y_{n+1}=y_n+\frac{1}{n+1}$

$y_2=2+\frac{1}{3},y_3=2+\frac{1}{3}+\frac{1}{4}$

$\dots y_n=2+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\dots\dots$

$y_{n+1}-y_n=\frac{1}{n+1}>0$

$y_{n+1}-y_n>0$

$y_{n+1}>y_n$

increasing sequence

48. Explanation:

$$\lim_{n\to\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+1}\right)$$

$$\lim_{n\to\infty}\frac{\left(n^2fn-\sqrt{n^2+1}\right)\left(\sqrt{n^2+n}+\sqrt{n^2+1}\right)}{\sqrt{n^2+n}+\sqrt{n^2+1}}$$

$$\lim_{n\to\infty}\frac{n^2+n-n^2-1}{\sqrt{n^2+n}+\sqrt{n^2+1}}$$

$$\lim_{n\to\infty}\frac{n^2+n-n^2-1}{\sqrt{n^2+n}+\sqrt{n^2+1}}$$

$$\lim_{n\to\infty}\frac{n-1}{\sqrt{n^2+n}+\sqrt{n^2+1}}=1$$

49. Explanation:

$$\lim_{\theta\to\frac{\pi}{2}}(1-5\cot\theta)^{\tan\theta}$$

At $\theta=\frac{\pi}{2}$ we get 1^∞ from

$$\lim_{\theta\to\frac{\pi}{2}}(1-5\cot\theta)^{\tan\theta}=e^{\lim_{\theta\to\frac{\pi}{2}}(\tan\theta(1-1-5\cot\theta))}$$

$$\lim_{\theta\to\frac{\pi}{2}}(1-5\cot\theta)^{\tan\theta}=e^{\lim_{\theta\to\frac{\pi}{2}}(\tan\theta(-5\cot\theta))}$$

$$\lim_{\theta\to\frac{\pi}{2}}(1-5\cot\theta)^{\tan\theta}=e^{-5}$$

50. Explanation:

$$b_n=\frac{(n+1)(n+2)\dots(n+n)}{n^n(n+1)(n+2)\dots(n+n)}$$

$$b_{n+1}=\frac{(n+1)(n+2)\dots2n(2n+1)+(2n+2)}{(n+1)^{n+1}}$$

$$\lim_{n\to\infty}\frac{b_{n+1}}{b_n}=\frac{\left(2+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{\left(2+\frac{1}{n}\right)^2\left(2+\frac{2}{n}\right)}=\frac{4}{e}$$

51. Explanation:

$$\lim_{n\to\infty}\frac{1}{n}\left(\prod_{k=1}^{n}(n+k)\right)^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{1}{n}((n+1)(n+2)\ldots(n+n))^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{n}{n}\left(\frac{(n+1)(n+2)\ldots(n+n)}{n^n}\right)^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right)^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=l\Rightarrow\lim_{n\to\infty}(a_n)^{\frac{1}{n}}=l$$

$$n=\frac{(n+1)}{n}\frac{(n+2)}{n}\ldots.\frac{(n+n)}{n}$$

$$\lim_{n\to\infty}\frac{a_n+1}{a_n}$$

$$\lim_{n\to\infty}\frac{(n+2)(n+3)\ldots(n+2)}{(n+1)^{n+1}}\times\frac{n^n}{(n+1)(n+2)\ldots(n+n)}$$

$$\lim_{n\to\infty}\frac{(2n+1)(2n+2)n^n}{(n+1)(n+1)^{n+1}}$$

$$\lim_{n\to\infty}\frac{(2n+1)(2n+2)}{(n+1)(n+1)}\left(\frac{n}{n+1}\right)^n$$

$$\lim_{n\to\infty}\frac{(2n+1)(2n+1)}{(n+1)(n+1)}\left(\frac{1}{1+\frac{1}{n}}\right)^n$$

$\frac{4}{e}$

52. Explanation:

$$\lim_{n\to\infty}\left(\frac{\left(a+\frac{1}{n}\right)\left(a+\frac{2}{n}\right)\left(a+\frac{3}{n}\right)\left(a+\frac{4}{n}\right)\ldots\ldots\left(a+\frac{k}{n}\right)}{a^k}\right)^n$$

$$\Rightarrow\lim_{n\to\infty}\left(1+\frac{1}{an}\right)\left(1+\frac{2}{an}\right)\ldots\ldots\ldots\left(1+\frac{k}{an}\right)$$

$$\Rightarrow\left(\frac{1}{a}\right)\left(\frac{2}{a}\right)\ldots\ldots\left(\frac{k}{a}\right)$$

$$\Rightarrow\frac{k!}{a^k}$$

53. Explanation:

Let $< a_n >$ & $< b_n >$ be two real sequences.

a. Let $\sum a_n=\sum b_n=\sum\frac{(-1)^n}{n}$

$\sum a_n\, b_n=\sum\frac{1}{n}$ is not convergent

$\Rightarrow$Statement a is correct

b. $\sum a_n$ is convergent

$\sum b_n$ is absolutely convergent

$\lim_{n\to\infty} a_n=0$

$\exists\, n\, c\, N$ s.t

$|a_n| < \epsilon \qquad \forall\, \cap \geq N$

$|a_nb_n|\leq|b_n|$

$\sum|b_n|$ is convergent

$\Rightarrow\sum|a_nb_n|$ is convergent

$\Rightarrow\sum a_nb_n$ is Absolutely convergent

$\Rightarrow\sum a_nb_n$ is convergent

Statement b & c

c. $a_n\geq 0\sum a_n$ convergent

$\{b_n\}^\infty{}_{n=1}$ & bounded

$|b_n|\leq m$

$|a_nb_n|\leq M|a_n|$

$\sum|a_n|$ is convergent

$\sum Ma_n$ is convergent

$\Rightarrow\sum a_nb_n$ is convergent

Statement c is correct

54. Explanation:

1. $f_n(x)=n^2x(1-x^2)^n \qquad x\in[0\ 1]$

   $\lim_{n\to\infty}f_n(0)=0$

   $\lim_{n\to\infty}f_n(1)=0$

   Let $0<x<1$

   $\lim_{n\to\infty}f_n(x)=\lim_{n\to\infty}n^2x(1-x^2)^n=0$

   $\Rightarrow\lim_{n\to\infty}f_n(x)=0$

   $\Rightarrow f_n(x)$ converges point wise

2. $\lim_{n\to\infty}\int_0^1 fn(x)dx=\lim_{n\to\infty}\int_0^1 n^2x(1-x^2)^n dx$

   $=\lim_{n\to\infty}n^2\int_0^1 x(1-x^2)^n dx$

   Put $x-x^2=t$

   $-2xdx=dt$

   $=\lim_{n\to\infty}n^2\int_0^1 t^n\left(\frac{-1}{2}\right)dt$

   $=\lim_{n\to\infty}\frac{-n^2}{2}\left[\frac{t^{n+1}}{n+1}\right]_0^1$

   $=\lim_{n\to\infty}\int_0^1 fn(x)dx=\lim_{n\to\infty}\frac{-n^2}{2(n+1)}\ DNE$

   $\int_0^1 f(x)=0$

   $\Rightarrow\lim_{n\to\infty}\int_0^1 fn(x)dx\neq\int_0^1 f(x)$

55. Explanation: $F_n(x)=n^2x(1-x)^n$

$\lim_{n\to\infty}F_n(x)=\lim_{n\to\infty}n^2x(1-x)^n$

For $0<x<1$

$0>-x>-1$

$1>1-x>0$

$0<1-x<1$

As $n\to\infty\ (1-x)^n\to 0$

$\lim_{n\to\infty}Fn(x)\to 0$ as $n\to\infty$

56. Explanation:

1. $F(x) = \lim\limits_{n\to\infty} \frac{x^2e^{nx}+x}{e^{nx}+1}$

$$= \lim_{n\to\infty} \frac{x^2 + \frac{x}{e^{nx}}}{1 + \frac{1}{e^{nx}}}$$

$F(x) = x^2$

$\Rightarrow F(x)$ is continuous

2. $\lim\limits_{x\to\infty} \frac{x^2e^{nx}+\cos x}{e^{nx}+1}$ $\qquad x \in \mathbb{R}$

$$\lim_{x\to\infty} \frac{x^2 + e^{nx}\cos x}{1 + e^{-nx}}$$

$$F(x) = \begin{cases} \cos x & x < 0 \\ \frac{1}{2} & x = 0 \\ x^2 & x > 0 \end{cases}$$

$F(x)$ is not continuous

3. $F(x) = \lim\limits_{n\to\infty} \log\left(x^n\left(1 + \frac{e^n}{z^n}\right)\right)^{\frac{1}{n}}$

$$= \lim_{n\to\infty}\left(\log x + \log(1 + \left(\frac{e}{x}\right)^n\right)^{\frac{1}{n}}$$

$$(x) = \begin{cases} 1 & 0 \le x < e \\ 1 & x = e \\ \log x & x > e \end{cases}$$

$\Rightarrow F(x)$ is continuous

57. Explanation:

1. $F(x) = \sum_{n=1}^{\infty} \frac{=g(x-n)}{2^n}$ $\qquad x \in \mathbb{R}$

As $g(x)$ is uniformly continuous then linear combination of $g(x)$ is also uniformly continuous

$\Rightarrow F(x)$ is U.C

2. $F(x) = \cos x \cos\left(\frac{\pi}{x}\right)$ $\qquad 2 \in (0,1)$

$$\lim_{x\to 1} \cos x \cos\left(\frac{\pi}{x}\right) = -\cos(1)$$

$$\lim_{x\to 0} \cos x \cos\left(\frac{\pi}{x}\right) = -DNE$$

$F(x)$ is not uniformly continuous

58. Explanation:

$$\sum_{n\ge 1} \frac{\sin(x/n)}{n}$$

$$\left|\frac{\sin(x/n)}{n}\right| \le \frac{\frac{x}{n}}{n} = \frac{x}{n^2}$$

$\Rightarrow \sum_{n\ge 1} \frac{\sin(x/n)}{n}$ converges uniformly

$\Rightarrow F(x)$ is continuous over IR

$$F'(x) = \sum_{n=1}^{\infty} \frac{\cos(x/n)}{n}$$

$$\left|\frac{\cos(x/n)}{n}\right| \le \frac{1}{n^2}$$

$\Rightarrow \sum \frac{\cos(x/n)}{n^2}$ is coverges uniformly

$\Rightarrow F'(x)$ is continuous

$\Rightarrow F\,(x)$ is differentiable over IR

59. Explanation: $Fn(x) = \frac{1}{1+nx}$

1. (0,1)

$x \in (0, 1)$

$$\lim_{n\to\infty} Fn(x) = \lim_{n\to\infty} \frac{1}{1+nx} = 0$$

$$\lim_{n\to\infty} Fn(x) = 0 = F(x)$$

$$Mn = sum|Fn(x) - F(x)|$$

$$= \sup_{x \in (0,1)} \left|\frac{1}{1+nx}\right|$$

$$\phi_n(x) = \frac{1}{1+nx}$$

Let $x = \frac{1}{n} \to 0$ as $n \to \infty$

$$Mn = \frac{1}{2} \neq 0$$

$\Rightarrow < Fn(x) >$ not converges uniformly.

[CUET PG]

[CUET PG 2022]

1. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if p is

(1) 2
(2) $\frac{1}{2}$
(3) $\frac{1}{3}$
(4) $\frac{1}{4}$

[CUET PG 2023]

2. The given series $\frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \cdots, (x > 0)$ is convergent in the interval

(1) $[1, \infty)$
(2) $(1, \infty)$
(3) $(0,1]$
(4) $[0,1)$

Answer Key	
1	**2**
1	2

[CU-CET]

[CUCET 2016]

1. The series $\sum_{n=1}^{m} \frac{n}{3.5.7...(2n+1)}$ converges to

(1) $\frac{1}{2}$
(2) $\frac{1}{3}$
(3) $\frac{1}{4}$
(4) $\frac{1}{5}$

[CUCET 2016]

2. The series $\sum_{k=2}^{m} \frac{1}{k(log log\ k)^{\alpha}}$ where α is a real no. and $log\ log\ k = k$

(1) Converges for all α
(2) Converges only for $\alpha \leq 0$
(3) Converges only for all α satisfying $0 < \alpha \leq 1$
(4) Converges only for all $\alpha > 1$

[CUCET 2016]

3. Let $\sum_{n=1}^{\infty} a_n x^n$be a power series. Determine which one of the following formulas does not define the radius of convergence ρ.

(1) $\rho = \frac{1}{|a_n|^{\frac{1}{n}}}$
(2) $\rho = \frac{1}{|na_n|^{\frac{1}{n}}}$
(3) $\rho = \frac{1}{\left|\frac{a_{n+1}}{a_n}\right|}$
(4) $\rho = \frac{1}{|a_n|^{\frac{1}{n}}}$

[CUCET 2016]

4. If power series $\sum_{n=1}^{\infty} a_n x^n$and $\sum_{n=1}^{\infty} b_n x^n$ have radius of convergence ρ_1, ρ_2 respectively, then the radii of convergence p of the power series$\sum_{n=1}^{\infty} (a_n + b_n) x^n$ is given by

(1) $\rho = \rho_1 + \rho_2$
(2) $\rho = \{\rho_1, \rho_2\}$
(3) $\rho = \{\rho_1, \rho_2\}$
(4) $\rho = |\rho_1 - \rho_2|$

[CUCET 2016]

5. Let $\sum_{n=1}^{\infty} a_n x^n$ be a power series with ρ its radius of convergence where $0 < \rho < \infty$. Determine which one of the following facts is correct.

(1) The series converges absolutely and uniformly on any closed interval in $(-\rho, \rho)$
(2) The series converges absolutely and uniformly on any subinterval of $(-\rho, \rho)$
(3) The series converges uniformly on $(-\rho, \rho)$
(4) The series converges absolutely on $[-\rho, \rho]$

[CUCET 2017]

6. Consider the statements

a. The series $\sum sin\ \frac{1}{n}$ is convergent

b. The series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \cdots$ is convergent.

Then

(1) both the statements (a) and (b) are true
(2) (a) is true and (b) is false
(3) (a) is false and (b) is true
(4) neither (a) nor (b) is true

[CUCET 2017]

7. The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ is convergent for

(1) all real values of x
(2) $|x| < 1$ only
(3) $|x| < 1$
(4) $-1 < x < 1$

[CUCET 2017]

8. If $u_n = \sqrt{n+1} - \sqrt{n}$ and $v_n = \sqrt{n^4 + 1} - n^2$, then

(1) $\sum_{n=1}^{\infty} u_n$ converges but $\sum_{n=1}^{\infty} v_n$ diverges
(2) $\sum_{n=1}^{\infty} u_n$ diverges but $\sum_{n=1}^{\infty} v_n$ converges
(3) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both converges
(4) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both diverges

[CUCET 2019]

9. The series $\sum_{n=1}^{\infty} \frac{n+1}{n^P}$ is convergent for

(1) $0 < p < 1$
(2) $1 < p < 2$
(3) $p = 2$
(4) $p > 2$

[CUCET 2019]

10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is

(1) convergent
(2) divergent
(3) conditionally convergent
(4) absolutely convergent

[CUCET 2019]

11. If a series $\sum_{n=1}^{\infty} a_n$ converges, then

(1) $a_n = 0$
(2) $a_n = \infty$
(3) $a_n = 1$
(4) $a_n = 10$

[CUCET 2019]

12. The radius of convergence of the series $1 + 2x + 3x^2 + 4x^2 + \ldots$ is

(1) 0
(2) 1
(3) ∞
(4) 2

[CUCET 2020]

13. $\sum_{n=1}^{\infty} \frac{n^{q+1}}{n^p}$ is convergent if

(1) $p - q \leq 1$
(2) $p - q = 1$
(3) $p - q \geq 1$
(4) $p - q > 1$

[CUCET 2020]

14. $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series, where $a_n = \frac{(-1)^n n}{2^n}$ and $b_n = \frac{(-1)^n}{log\,(n+1)}$ for all $n \leq N$. Then

(1) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are absolutely convergent
(2) $\sum_{n=1}^{\infty}$ a is absolutely convergent but $\sum_{n=1}^{\infty} b_n$ is conditionally convergent
(3) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent but $\sum_{n=1}^{\infty} b_n$ is absolutely convergent
(4) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are conditionally convergent

[CUCET 2021]

15. Which one of the following series is convergent?

(1) $\sum_{n=1}^{\infty} n^2$
(2) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
(3) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$
(4) $\sum_{n=1}^{\infty} n^3$

[CUCET 2021]

16. Let $x \in R$. The set of all x at which the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ converges is

(1) [-1, 1]
(2) (-1,1)
(3) R
(4) [0,1]

Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	4	4	3	1	3	4	2	4	3
11	**12**	**13**	**14**	**15**	**16**				
1	2	4	2	2	3				

[PRACTICE QUESTIONS]

1. The series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is

(1) convergent if $p > 1$ and divergent if $0 < p \leq 1$.
(2) convergent if $0 < p < 1$ and divergent if $p \geq 1$.
(3) convergent $\forall\, p$.
(4) divergent $\forall\, p$.

2. The test for convergence of an alternating series was given by

(1) Cauchy
(2) D'Alembert
(3) Raabe
(4) Leibnitz

3. The infinite series $1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$ to ∞

(1) $p < 1$
(2) $p = 1$
(3) $p \leq 1$
(4) $p > 1$

4. The infinite series expansion of $\log(1 + x)$ is valid for

(1) $x > -1$ only
(2) $x < 1$ only
(3) $|x| < 1$ only
(4) $-1 < x \leq 1$

5. Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences of real numbers such that $a_n = b_n - b_{n+1}$ for $n \in N$. If $\sum b_n$, is convergent, then which of the following is true?

(1) $\sum a_n$ may not converge
(2) $\sum a_n$ is convergent and $\sum a_n = b_1$
(3) $\sum a_n$ is convergent and $\sum a_n = 0$
(4) Za, is convergent and $\sum a_n = a_n = a_1 - b_1$

6. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{1}{2^n} x^{3n}$ is

(1) 1
(2) ∞
(3) 1/2
(4) $2^{1/3}$

7. Which one of the following is true?

(1) If $\sum a_n$ diverge and $a_n > 0$, then $\sum \frac{a_n}{1+a_n}$ diverges
(2) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ diverges
(3) $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ converges
(4) If $\sum a_n$ converges and $\sum b_n$, diverges, then $\sum(a_n + b_n)$ converges

8. If $\sum a_n = A. \sum |a_n| = B$ and A and B are finite, then

(1) $|A| = B$
(2) $A \leq B$
(3) $|A| \geq B$
(4) $A = B$.

9. Which one of the following series converges?

(1) $\sum \cos \frac{1}{n^2}$
(2) $\sum \sin \frac{1}{n^2}$
(3) $\sum \frac{1}{n^{1+1/n}}$
(4) $\sum n^{\cos 3}$

10. The sum of the series $\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$ is

(1) $\frac{\pi^2}{8}$
(2) $\frac{\pi^2}{8}$
(3) $\frac{\pi}{8}$
(4) 1

11. The series $x+\frac{2^2}{2!}+\frac{3^3x^3}{3!}$ is convergent if x belongs to the interval
(1) (0,1/e)
(2) $(1/e,\infty)$
(3) $(2/e,3/e)$
(4) $(3/e,4/e)$

12. The series $\sum_{k=0}^{\infty}\frac{k^2+3k+1}{(k+2)!}$ Converges to
(1) 1
(2) 1/2
(3) 2
(4) 3

13. We know that $xe^x=\sum_{n=1}^{\infty}\frac{x''}{(n-1)!}$ These series $\sum_{n=1}^{\infty}\frac{2''n^2}{n!}$ Converges to
(1) e^2
(2) $2e^2$
(3) $4e^2$
(4) $6e^2$

14. The series $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$
(1) diverges
(2) converges to 1
(3) converges to $\frac{1}{2}$
(4) converges to $\frac{1}{7}$

15. Consider the series
$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(\sqrt{n+1}-\sqrt{n+1})$
Then
(1) the series is convergent but not absolutely convergent
(2) the series is divergent
(3) the nth term of series does not converge to zero
(4) the series is absolutely convergent

17. If $f(x)=\lim_{n\to\infty}S_n(x)$, where $S_n(x)=\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+\dots+\frac{x}{(nx+1)((n+1)x+1)}$ then the function f is
(1) continuous nowhere.
(2) continuous everywhere.
(3) continuous everywhere except at countably many points.
(4) continuous everywhere except at one point.

18. If the series $\sum_{n=0}^{\infty}(sinx)$^$n$ converges to the value $(4+2\sqrt{3})$ for some value of x in $\left(0,\frac{\pi}{2}\right)$, then the value of x is
(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{5}$
(4) $\frac{\pi}{6}$

19. Consider the statements
$S_1: 1-1+1-1+1-1+\cdots=\pm1.$
$S_2: \frac{1}{1+2}=1-2+2^2-2^3+\cdots Then$
(1) S_1 is true bit $S_2$is false
(2) S_1 is false but S_2 is true
(3) both S_1 and S_2 are true
(4) both $S_1$and $S_2$are false

20. Let $\{x_n\}$ be a sequence of positive real numbers. Then which of the following is false
(1) If $\sum_{n=1}^{\infty}x_n$ is convergent then $\sum_{n=1}^{\infty}\sqrt{x_n}$ is convergent.
(2) If $\sum_{n=1}^{\infty}x_n$ is convergent then $\sum_{n=1}^{\infty}x_n^2$ is convergent.
(3) If $\sum_{n=1}^{\infty}x_n^2$ is convergent then $\lim_{n\to\infty}x_n=0$.
(4) If $\sum_{n=1}^{\infty}\sqrt{x_n}$ is convergent then $\lim_{n\to\infty}x_n=0$.

21. Given S_1 and S_2, where
S_1: A series $\sum_{n=0}^{\infty}a_n$ converges if for a given $\in>0$ there exists $N_0\in\mathbb{N}$ such that $|a_n+1-a_n|<\in$ $for\ all\ n\geq N_0$.
S_2: A series $\sum_{n=0}^{\infty}a_n$ converges if $|a_{n+1}-a|<\alpha^n$ where α is a fixed real number in $(0,1)$,
Which of the following statements are true?
(1) S_1 is true but S_2 is false.
(2) S_1 is false but S_2 is true.
(3) Both S_1 and S_2 are true.
(4) Both, S_1 and S_2 are false.

22. The sum of the series $\sum_{n=1}^{\infty}[(n+1)]^{\frac{1}{2}}-n^{\frac{1}{2}}]\ is$
(1) less than-1
(2) equal to-1
(3) greater than-1 but less than 2
(4) none if the above

23. $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{2k}{k^2+n^n}=$
(1) 0
(2) log 2
(3) 2
(4) ∞

24. Which of the following statements are true?
(1) $\sum_{n=0}^{\infty}\log\frac{n}{n^{\frac{3}{2}}}$ is convergent
(2) $\sum_{n=0}^{\infty}\frac{n^2}{n!}$ is divergent
(3) $\sum_{n=0}^{\infty}\frac{1}{n\log n}$ is convergent
(4) $\sum_{n=0}^{\infty}\frac{e^n}{n}$ [illegible]vergent

25. The largest interval I such that the series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ converges whenever $x \in I$ is equal to

(1) $[-1.1]$.
(2) $[-1,1]$
(3) $[-1,1]$
(4) $(-1,1)$

26. Let $\sum a_n$ be convergent series. Let $b_n = a_n + 1 - a_n$ for all $n \in \mathbb{N}$. then

(1) $\sum b_n$ should also be convergent and $(b_n) \to 0$ *as* $n \to \infty$.
(2) $\sum b_n$ need not be convergent but $(b_n) \to 0$ *as* $n \to \infty$.
(3) $\sum b_n$ is convergent but (b_n) need not tend to zero as $n \to \infty$.
(4) None of the above statements is true.

27. Consider the real sequences (a_n) and (b_n) such that $\sum a_n b_n$ converges. Which of the following statements is true ?

(1) If $\sum a_n$ an converges, then (b_n) is bounded.
(2) If $\sum b_n$ an converges, then (a_n) is bounded.
(3) If (a_n) is bounded, then (bn) converges.
(4) If (an) is unbounded, then (bn) bounded.

28. The series $\sum_{n=1}^{\infty} \log cos\left(\frac{1}{n}\right)$

(1) converges absolutely
(2) converges conditionally but not absolutely
(3) diverges to +∞
(4) diverges to - ∞

29. Consider the series S_1 and S_2 given by

$S_1 \coloneqq \sum_{n=1}^{\infty}\left(1-\frac{1}{2^n}\right)^n$

$S_2 \coloneqq \sum_{n=1}^{\infty}\frac{(-1)^n log^3 n}{n}$

Which one of the following is true"

(1) S_1 converges and S_2 converges absolutely
(2) S_1 converges and S_2 converges conditionally but not absolutely
(3) S_1 converges and S_2 diverges
(4) S_1 diverges and S_2 converges

30. Consider a series $\sum_{n=1}^{\infty} a_n$ of real numbers. Which of the following is necessarily true?

(1) If $|a_n| \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges conditionally but it does not necessarily converge absolutely.
(2) If $a_n \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges conditionally.
(3) If $\frac{1}{n^2} \leq a_n \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges absolutely.
(4) If $0 \leq a_n \leq \frac{n^2+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges absolutely.

31. The sum $\frac{1}{1001}+\frac{1}{1002}+\cdots+\frac{1}{2000}$, is:

(A) less then 1.
(B) more then $\frac{1}{2}$.
(C) more then log2
(D) less then log2

Which of the above assertions are correct?

(1) A < B and C only
(2) A, B and D only
(3) A and C only
(4) B and C only

32. Let $\sum_{n=0}^{\infty} a_n X^n$ be a power series with coefficients and of convergence R such that $0 < R < \infty$. consider the following assertions:

(A) If $\sum_{n=0}^{\infty} a_n x^n$ converges for some x with $|x| = R, then \sum_{n=0}^{\infty} a_n x^n$ converges for every $x\ with\ |x| = R$
(B) If $|x| > R$, then $\sup_{k \geq 10} \sum_{n=1}^{k} |a_n||x|^n = \infty$.
(C) $\sum_{n=0}^{\infty} a_n x^n$ diverges for some x with $|x| = R$, then $\sum_{n=0}^{\infty} a_n x^n$ diverges for every x with $|x| = R$.
(D) Let $S_k(x) = \sum_{n=2}^{k} a_n x^n$ for all $k \geq 2$. Then, $\{S_k(x)\}_{k=2}^{\infty}$ is a Cauchy sequence for every $x \in (-R, R)$.

Which of the above assertions is / are correct?

(1) A and C only
(2) B and D only
(3) D only
(4) B only

33. Let $f : [-2021, 2021] \to \mathbb{R}$ be a function such that $|f(x) - f(y)| \leq 2021\,|x-y|$ for all $x, y \in [-2021, 2021]$ and further $f(20) = 21$ and $f(21) = 20$. Consider the following statements:

A. f is a continuous function.
B. There is an x with $-41 < x < 41$ such that $f(x) = x$.

Then

(1) Both A and b are true.
(2) B is true and A is false.
(3) Both A and B are false.
(4) A is true and B is false.

34. Which of the following series is convergent:

(1) $\sum_2^\infty \frac{1}{n\, lod\, n}$

(2) $\sum_2^\infty \frac{1}{n(lod\, n)^2}$

(3) $\sum_1^\infty cos n$

(4) $\sum_1^\infty cos \frac{1}{n^2}$

35. Find the sum of the series

$\sum_{k=1}^{\infty} \frac{k^2}{k!}$

(1) 1e

(2) 2e

(3) 4e

(4) 6e

36. What is the radius of convergence of the power series

$\sum_{k=1}^{\infty} \sqrt{\log k}\ x^k$?

(1) 1

(2) 2

(3) 4

(4) 6

37. What are the values of $a \in \mathbb{R}$ for which the following series is convergent?

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^a}$.

(1) $\propto > 1$

(2) $\propto = 1$

(3) $\propto \geq 1$

(4) $\propto \leq 1$

38. Evaluate:

$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k^2}}$.

(1) $\ln|\sqrt{2}+1|$

(2) $\ln|2+1|$

(3) $\ln|\sqrt{2}-1|$

(4) $\ln|\sqrt{2}+2|$

39. Evaluate:

$\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n} \cos\left(\frac{\pi k}{2n}\right)$.

(1) $\frac{6}{\pi}$

(2) $\frac{4}{\pi}$

(3) $\frac{8}{\pi}$

(4) $\frac{2}{\pi}$

40. Find the interval of convergence of the series:

$(x+1) - \frac{(x+1)^2}{4} + \frac{(x+1)^3}{9} - \frac{(x+1)^4}{16} + \cdots$

(1) (2,0)

(2) (-4,0)

(3) (-2,0)

(4) (-2,2)

41. For what values of p does the following series converge?

$1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$

(1) $P > 0$

(2) $P = 0$

(3) $P < 0$

(4) $P \geq 0$

42. Which of the following series is divergent?

(1) $\sum_{n=1}^{\infty} \sqrt{\frac{2n^2+3}{5n^3+7}}$

(2) $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+\frac{3}{2}}}$

(3) $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$.

(4) None of the above

43. Find the interval of convergence of the series:

$\sum_{n=1}^{\infty} \frac{\log(n+1)}{\sqrt{n+1}} (x-5)^n$

(1) (4,6)

(2) (2,4)

(3) (8,10)

(4) (6,8)

45. Determine if each of the following series is absolutely convergent, conditionally convergent

(1) $\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, x \in \mathbb{R}$

(2) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$

(3) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2n+3}$

(4) None of the above

46. Pick out the true statements:

(1) If the series $\sum_n a_n$ and $\sum_n b_n$ are convergent, then $\sum_n a_n b_n$ is also convergent.

(2) If the series $\sum_n a_n$ is convergent and if $\sum_n b_n$ is absolutely convergent, then $\sum_n a_n b_n$ is absolutely convergent.

(3) If the series $\sum_n a_n$ is convergent, $a_n \geq 0$ for all n, and if the sequence $\{b_n\}$ is bounded, then $\sum_n a_n b_n$ is absolutely convergent.

(4) None of the above

47. Test the following series for convergence:

(1) $\sum_{n=1}^{\infty} \left(n^{\frac{1}{n}} - 1\right)^n$

(2) $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$

(3) both (1) and (2)

(4) None of the above

where $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive term

48. In each of the following verify whether the series is not absolutely convergent:

(1) $\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{n}{n+1}}$

(2) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \sin\frac{1}{n}$

(3) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{(n+1)(n+2)}$

(4) None of the above

49. Sum the following infinite series:

$\frac{1}{6} + \frac{5}{6.12} + \frac{5.8}{6.12.18} + \frac{5.8.11}{5.12.18.24} + \cdots$

(1) 1

(2) 2

(3) 4

(4) 6

50. Test the following series for convergence:

(1) $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right);$

(2) $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\cdots$

(3) both (1) and (2)

(4) None of the above

51. Pick out the convergent series.

(1) $\sum_{n=1}^{\infty}\frac{(n+1)^n}{n^{n+\frac{3}{2}}}$

(2) $\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$

(3) Bothe (1) & (2)

(4) None of the above

52. Let $a>0$. Find the sum of the infinite series:

$$1+\frac{(\log a)^2}{2!}+\frac{(\log a)^4}{4!}+\frac{(\log a)^6}{6!}+\cdots$$

(1) $\frac{1}{2}\left(\frac{1}{a}+a\right)$

(2) $\frac{1}{2}\left(a-\frac{1}{a}\right)$

(3) $\frac{1}{2}\left(a+\frac{1}{2}\right)$

(4) $\frac{1}{2}\left(a+\frac{1}{a}\right)$

53. Which of the following series are convergent?

(1) $\sum_{n=1}^{\infty}\sqrt{\frac{2n^2+3}{5n^3+1}}.$

(2) $\sum_{n=1}^{\infty} n2\,x(1-x^2)^n$, where $0<x<1$.

(3) Both (1) & (2)

(4) None of the above

54. In each of the following cases, state whether the given series is divergent.

(1) $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\cdots$

(2) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+1}.$

(3) $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(n-1)}.$

(4) None of the above

Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	4	3	2	4	1	2	2	1
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	3	4	1	4	-	4	1	3	1
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	3	2	1	1	1	4	1	4	3
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
2	2	3	2	2	1	3	1	4	3
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1	4	1	-	4	3	3	1	1	3
51	**52**	**53**	**54**						
1	4	2	2						

EXPLANATIONS

[CUET PG]

1. **Explanation:**

$\sum \frac{1}{n^p}$ converges if $P > 1$

2. **Explanation:**

$\frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \cdots$

$\sum_{n=1}^{\infty} \frac{x}{n(n+2)} = \frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \cdots$

$a_n = \frac{x^n}{n(n+2)}$

$\frac{1}{R} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{x^{n+1}}{(n+1)(n+3)} \times \frac{n(n+2)}{x^n}$

$= \lim_{n\to\infty} x \frac{n(n+2)}{(n+1)(n+3)}$

$= x$

$|x| < 1$

$R = 1$

Radius of convergent is 1 & $x > 0$

$(0,1]$

[CU-CET]

1. **Explanation:**

$\sum_{n=1}^{\infty} \frac{n!}{3.5.7 \ldots (2n+1)}$

Let $u_{n+1} = \frac{1.2.3 \ldots n(n+1)}{3.5.7 \ldots (2n+1)(2(n+1)+1)}$

$\frac{u_{n+1}}{u_n} = \frac{n+1}{2n+3} = \frac{n(1+1/2)}{n(2+3/n)} \Rightarrow \lim_{n\to 0} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{1+1/n}{2+3/n} = 1/2$

$\Rightarrow \sum u_n$ is convergent

$\Rightarrow$ option A is correct

2. **Explanation:**

$\sum_{n=1}^{\infty} \frac{1}{K(\log K)^{\alpha}}$

$\therefore$ we know that given series is convergent only for all $\alpha > 1$

$\Rightarrow$ option (4) is correct

$\sum_{n=1}^{\infty} n^{2n} x^n$

$a_n = n^{2n}$

$\frac{1}{R} = \lim_{n\to\infty} \sup |a_n|^{1/n}$

$\lim_{n\to\infty} \sup |n^{2n}|^{1/n}$

$\lim_{n\to\infty} \sup |n^2|^{1/n}$

$\frac{1}{R} = \infty$

$R = 0$

$\Rightarrow$ option (1) is correct

3. **Explanation:**

$\rho = \frac{1}{\lim \inf |a_n|^{1/n}}$ formulas does not define the radius of convergence ρ

$\Rightarrow$ option (4) is correct

4. **Explanation:**

$\sum_{n=1}^{\infty} a_n x^n \sum_{n=1}^{\infty} b_n x^n$ has radius of convergence ρ_1, ρ_2 respectively $\sum_{n=1}^{\infty} (a_n + b_n) x^n$ has radius of converges is

$\min\{\rho_1, \rho_2\}$

$\rho = \min\{\rho_1, \rho_2\}$

$\Rightarrow$ option (3) is correct

5. **Explanation:**

(1) The series converges absolutely and uniformly on any closed interval in $(-\rho, \rho)$

6. **Explanation:**

(a) $\sum a_n = \sum \sin\frac{1}{n}$ $\qquad a_n = \sin\frac{1}{n}$

Let $\sum = \frac{1}{n}$ $\quad b_n = \frac{1}{n}$

By comparison test

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin 1/n}{1/n}$

$= 1 \neq 0$

Then $\sum a_n$ & $\sum b_n$ behave same

$\therefore \sum a_n = \sum \sin\frac{1}{n}$ is also divergent

Now (b) $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \cdots$

$\sum a_n = \sum_{n=1}^{\infty} \frac{(2n-1)(2n)}{(2n+1)^2(2n+2)^2}$

$a_n = \frac{(2n-1)(2n)}{(2n+1)^2(2n+2)^2}$

Let $\sum b_n = \sum \frac{1}{n^2}$ $\qquad b_n = \frac{1}{n^2}$

By comparison test $\lim_{n\to\infty} \frac{a_n}{b_n} = l \neq 0$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(2n-1)(2n)}{(2n+1)^2(2n+2)^2} \times \frac{n^2}{1}$

$= \lim_{n\to\infty} \frac{n^4\left(2-\frac{1}{n}\right)(2)}{n^4\left(2+\frac{1}{n}\right)^2\left(2+\frac{2}{n}\right)^2}$

$= \frac{2\times 2}{4\times 4} = \frac{1}{4} \neq 0$

$\Rightarrow \sum a_n$ & $\sum b_n$ behave same

But $\sum b_n = \sum \frac{1}{n^2}$ is cgt

So $\sum a_n$ is cgt

$\Rightarrow$ I is not true II is true

$\Rightarrow$ option (3) is correct